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Political Economy Of Transparency

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Abstract
This dissertation studies public policy in coordination environments, where there is complementarity in the agents' actions. The first chapter studies a model of currency attacks in which the government can choose a credible signal about the fundamentals of the economy. Public signals create partial common knowledge that can lead to multiple equilibria. The optimal policy with commitment is characterized when, if there is multiplicity, the government only cares about its lowest equilibrium payoff. In this case, the public signal is informative and leads to a unique equilibrium, which is preferred to a full disclosure policy. Our results indicate that the government has incentives for being vague in its communication. The highest equilibrium payoff for the government can be achieved with a two-signal policy. In equilibrium, agents follow the public signal and take the same action: either there is a coordinated attack, or all speculators refrain from attacking. The second paper develops a model where short-term reputation concerns guide the public disclosure of information. There are and high and low states that determine the productivity of investment, and the high state is more likely if the government is efficient rather than inefficient. Governments know the state and make public reports with the objective to be perceived as efficient. I find that the inefficient government is never completely truthful in equilibrium. When the efficient government is truthful, the inefficient government sends false reports of a high state with positive probability. This creates uncertainty following the report of a high state: if the true state is high, productivity is underestimated; if the true state is low, productivity is overestimated.

This bias reduces welfare in the high state, but there is a tradeoff in the low state: marginal entrepreneurs lose from overestimating productivity; all entrepreneurs gain from a higher aggregate investment. I show that when the trust in the government's report is low, the inefficient government can improve welfare in the low state by sending false reports that increase investment. However, as the trust in the false reports rises, the bias in entrepreneurs' beliefs becomes large and welfare decreases.

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ABSTRACT

POLITICAL ECONOMY OF TRANSPARENCY

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This dissertation studies public policy in coordination environments, where there is complementarity in the agents' actions. The first chapter studies a model of currency attacks in which the government can choose a credible signal about the fundamentals of the economy. Public signals create partial common knowledge that can lead to multiple equilibria. The optimal policy with commitment is characterized when, if there is multiplicity, the government only cares about its lowest equilibrium payoff. In this case, the public signal is informative and leads to a unique equilibrium, which is preferred to a full disclosure policy. Our results indicate that the government has incentives for being vague in its communication. The highest equilibrium payoff for the government can be achieved with a two-signal policy. In equilibrium, agents follow the public signal and take the same action: either there is a coordinated attack, or all speculators refrain from attacking. The second paper develops a model where short-term reputation concerns guide the public disclosure of information. There are and high and low states that determine the productivity of investment, and the high state is more likely if the government is *efficient* rather than *inefficient*. Governments know the state and make public reports with the objective to be perceived as efficient. I find that the inefficient government is never completely truthful in equilibrium. When the efficient government is truthful, the inefficient government sends false reports of a high state with positive probability. This creates uncertainty following the report of a high state: if the true state is high, productivity is underestimated; if the true state is low, productivity is overestimated. This bias reduces welfare in the high state, but there is a tradeoff in the low state: marginal entrepreneurs lose from overestimating productivity; all entrepreneurs gain from a higher aggregate investment. I show that when the trust in the government's report is low, the inefficient government can improve welfare in the low state by sending false reports that increase investment. However, as the trust in the false reports rises, the bias in entrepreneurs' beliefs becomes large and welfare decreases.

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CHAPTER 1 : CURRENCY ATTACKS AND GOVERNMENT COMMUNICATION

with Felipe Shalders

1.1. Introduction

Informed governmental agencies are often criticized for the poor quality of the information they release. Referring to the early years of Alan Greenspan as head of the Fed, Blinder and Reis (2005) write:

> *Soon Greenspan, who is far from plainspoken in any case, became known for such memorable phrases as 'mumbling with great incoherence'- which he used (with a hint of humor) to characterize his own version of Fedspeak.*

In this paper, we argue that it is optimal for the government to be vague in its communication. This happens because government's preferences do not coincide with preferences of other economic agents. When the government has access to payoff relevant information, it needs to be vague in order to induce agents to take the government's most preferred action.

We analyze the environment where a government can release a public signal about the fundamentals of the economy. In our model, the government would like to maintain a currency peg. The peg can be attacked by a continuum of speculators, who wish to profit from a currency devaluation. Payoffs depend on the state of fundamentals of the economy, the action taken by speculators, and the government's choice between defending or abandoning the peg. If fundamentals are weak (low states), speculators can have large profits from attacking the currency and the government has to pay a high cost to maintain the peg; if fundamentals are strong (high states), speculators can have at most small profits from attacking the currency and the cost of defending the peg is low. The cost of maintain the peg is increasing in the number of speculators that attack the currency.

Following Morris and Shin (1998), we assume that speculators receive noisy private signals about the fundamentals. Thus, if public signals are imprecise, one could expect them to have small effects on speculators beliefs about the state of fundamentals. This raises the question: why are vague announcements effective?

When information is dispersed across speculators, an imprecise signal about the fundamentals can have large effects because it changes the beliefs of a speculator about what other speculators believe. If the government can delegate to an informed and independent agency (such as the Fed) the mission to send a public signal (such as the FOMC statements) about the state of fundamentals, this public signal generates partial common knowledge about the unknown state. Thus, government communication induces coordination among speculators, even if the public signal has a low precision.

In our model, the government chooses an arbitrary partition of the space of fundamentals. The public signal reveals in which element of the partition the true fundamentals lie. Only truthful signals are allowed. Given the common prior and the private and public signals, speculators use Bayesian updating and then decide whether to attack the currency or not.

In a model where the state of fundamentals is common knowledge, multiple equilibria arise because of the coordination problem faced by speculators. However, Morris and Shin (1998) show that the introduction of noisy private signals about the fundamentals leads to a unique equilibrium, where speculators use cutoff strategies based on their private signals. Our introduction of public signals breaks the uniqueness result in Morris and Shin (1998).[1] To characterize the optimal disclosure policy we thus assume that, in the case of multiplicity, the government only cares about the worst equilibrium outcome.[2] Under this assumption,

[1] In a different setting, Angeletos et al. (2006) study a model where policy interventions generate endogenous information, leading to multiple equilibria. We, however, assume that the government can commit ex-ante to a disclosure policy. If we remove this assumption, our results change significantly. See Section 1.5.

[2] There are two ways to justify this assumption. First, this selection mechanism maximizes speculators' payoffs. Second, we take Morris and Shin (1998) as a benchmark to ask whether the government is better off by sending an informative signal. The optimal signal derived from our equilibrium selection provides a strictly positive lower bound for the government's benefits from sending a public signal.

we show that the optimal disclosure policy is, without loss of generality, a policy with two signals (a two interval partition). We interpret this result as a deliberate decision from the government to be vague - indeed, if the government could reveal the exact state of the economy, it would choose not to do so.

We then move to a characterization of the optimal disclosure policy. Two signals are sent when the optimal policy is implemented: a low one, corresponding to *bad* states of fundamentals (a coordinated attack in this region is always profitable), and a high one, for the *not too bad* states (a coordinated attack is not always profitable). We find that the government ‘’hides” some intermediate states with strong ones in the *not too bad* region. Intuitively, this is the optimal signal because, after observing a high signal, speculators assign a sufficiently high probability to states where it is not profitable to attack, which allows the government to prevent attacks in intermediate states. In order to do this, the government commits to acknowledging the really bad states of fundamentals.

We find that the subgame that follows the optimal disclosure policy has a unique equilibrium. After observing the low signal, speculators coordinate on attacking and the government abandons the currency peg. When a high signal is observed, speculators refrain from attacking and the peg is maintained. If the government had included too many states in the *not too bad* region, this would have lead to an equilibrium with currency attacks after the high signal, which the government wishes to avoid. In other words, the government wants to minimize the revelation of bad states by reducing the *bad* region up to the limit where not attacking after observing the high signal is still the unique possible action to be taken in equilibrium.

The final result of this paper is that commitment is essential for the government to benefit from disclosing information. When the government cannot commit to a disclosure policy, there exist equilibria in which the government is made worse off by sending a public signal. Without commitment, the government wishes to fully reveal the good states, which allows the speculators to coordinate on attacking in bad and intermediate states.

Related literature.

This paper is related to the literature on self-fulfilling currency crises when payoffs are not common knowledge among speculators. The idea that *small* deviations from common knowledge can have a large impact on equilibrium outcomes dates back at least to Rubinstein's mail game (Rubinstein (1989)), and has gained great attention since Carlsson and van Damme (1993) and Morris and Shin (1998).

We build on the model of Morris and Shin (1998) to introduce a public signal that generates partial common knowledge. In different settings, the interaction between public and private signals in coordination games has been studied in Morris and Shin (2002),Morris and Shin (2003), Hellwig (2002).[3]

In our model, public signals induce coordination among speculators, as in Angeletos et al. (2006), Angeletos and Pavan (2007, 2009) and Angeletos and Pavan (2013). Breaking the uniqueness result in Morris and Shin (1998), Angeletos et al. (2006) point out that policy interventions that convey some information about the fundamentals may lead to multiple equilibria. We focus on optimal government communication, thus policy in our model is the revelation of information itself, and, as opposed to the literature, does not change payoff relevant parameters. The government has incentives to release information about the fundamentals in order to influence the final outcome of the game. This is true even if, by restoring partial common knowledge, the game that follows the government's decision admits multiple equilibria.

The paper also relates to the literature on coordination motives in information acquisition (e.g., Hellwig and Veldkamp (2009), Myatt and Wallace (2012)). Hellwig and Veldkamp (2009) show that, when there are complementarities in the actions, agents "*want to know*

[3]In those papers, the public signal cannot generate common knowledge about dominance regions. In Morris and Shin (1998), this lack of common knowledge is important for equilibrium uniqueness. In our model, we allow for the public signal to make such revelations.

what others know". In line with their findings, our equilibrium displays speculators that coordinate on the public signal and take the same action regardless of their private information.

Finally, the paper relates to the literature on Bayesian persuasion (e.g., Kamenica and Gentzkow (2011)), which studies the optimal signal structure from the perspective of a sender who wants to influence a rational Bayesian receiver to take the sender's preferred action. This is done by affecting the receiver's beliefs. In addition to this effect, our model also takes into account the interaction of speculators who have private information, where a public signal can play an important role on coordination. The optimal policy is designed to maximize the probability that speculators coordinate on not attacking the currency peg.

Structure of the paper. The remainder of this paper is divided as follows. In Section 1.2, a motivating example is presented. Section 1.3 presents the model and some of its equilibrium properties. The main results are described in Section 1.4. Section 1.5 analyzes the model without the commitment assumption and Section 1.6 concludes the paper. A numerical example is provided in Appendix A.1, and the proofs that are omitted in the main text are presented in Appendix A.2.

1.2. A simple example

Before the full model is introduced, we present an example that that conveys the main ideas in this paper. It illustrates the effects of adding a public signal to a model of currency attacks in which speculators observe a private signal about the fundamentals.

Consider an economy where the state of the fundamentals is given by $\theta \in \Theta = \{0, 0.5, 1\}$, and the common prior assigns the same probability for each state. There is a continuum of speculators with unit mass. The government initially pegs the exchange rate at 1, and the equilibrium rate without intervention is θ. Speculators decide simultaneously whether to attack the currency peg or not. Given θ and the size of the attack, the government decides whether to abandon the peg or defend the currency.

Each speculator pays a cost of $t = 0.4$ to attack the currency peg, and the gross payoff is $1 - \theta$ if the peg is abandoned (a successful attack). The speculator's net payoff from a successful attack is thus given by

$$u(\theta) = \begin{cases} 0.6, & \text{if } \theta = 0, \\ 0.1, & \text{if } \theta = 0.5, \\ -0.4, & \text{if } \theta = 1, \end{cases}$$

and the payoff from an unsuccessful attack is -0.4. If the speculator refrains from attacking, his payoff is 0.

The government derives a value $v = 1$ from the currency peg, and the cost of defending it is given by $c(\theta, \alpha) = 1.3 - \theta + \alpha$, where α is the mass of speculators who attack the peg. If the government abandons the peg, its payoff is 0. The critical mass of attackers necessary for the government to abandon the peg is given by

$$a(\theta) = \begin{cases} 0, & \text{if } \theta = 0, \\ 0.2, & \text{if } \theta = 0.5, \\ 0.7, & \text{if } \theta = 1, \end{cases}$$

that is, if θ is the state and α is the fraction of speculators who attack the currency, then the government abandons the peg if $\alpha > a(\theta)$, and defends the peg if $\alpha < a(\theta)$.

1.2.1. Common knowledge

If the state of the fundamentals is common knowledge, the game admits two equilibria. In one equilibrium there is a coordinated attack on the currency peg after $\theta = 0.5$ is observed, which forces the government to abandon the peg. The other equilibrium features no attack after $\theta = 0.5$ is observed, and the government chooses to maintain the peg. In both equilibria there is no attack when $\theta = 1$, since the speculators know that it is not profitable to attack; and every speculator attacks the currency when $\theta = 0$, since they know that the the government will abandon the peg regardless of the size of the attack.

1.2.2. Private signal

Suppose that the true state is unknown, but the speculators observe a private signal $x \in \{0, 0.5, 1\}$, with conditional probability $\mathbb{P}(x|\theta)$ as follows:

	$x = 0$	$x = 0.5$	$x = 1$
$\mathbb{P}(x\|\theta = 0)$	.50	.25	.25
$\mathbb{P}(x\|\theta = .5)$	.25	.50	.25
$\mathbb{P}(x\|\theta = 1)$	.25	.25	.50

As in Morris and Shin (1998), this game admits a unique equilibrium. In equilibrium, speculators attack the currency if the private signal is $x \leq 0.5$, and the government abandons the peg if $\theta \leq 0.5$.[4]

1.2.3. Private and public signals

Now suppose that the government can commit to a disclosure policy as follows. First, the government partitions the state space Θ, and then it sends a public signal y that reveals in which element of the partition the true fundamentals lie. The introduction of a public

[4] The proof of is constructed in 6 steps:

1. If $\theta = 0$, the government abandons the peg in equilibrium regardless of the size of the attack.
2. If $x = 0$, since the peg is abandoned when $\theta = 0$, the equilibrium payoff from attacking is at least $\mathbb{P}(\theta = 0|x = 0) - 0.4 = 0.1$; thus, speculators attack when $x = 0$.
3. If $\theta = 0.5$, since speculators attack when $x = 0$, the size of the attack is at least $\mathbb{P}(x = 0|\theta = 0.5) = 0.25 > a(0.5) = 0.2$: thus, the government abandons the peg when $\theta = 0.5$.
4. If $x = 0.5$, since the peg is abandoned when $\theta \leq 0.5$, the equilibrium payoff from attacking is at least $\mathbb{P}(\theta = 0|x = 0.5) + 0.5\mathbb{P}(\theta = 0.5|x = 0.5) - 0.4 = 0.25 + 0.5^2 - 0.4 = 0.1$: speculators attack.
5. If $x = 1$, since the payoff from attacking when $\theta = 1$ is -0.4 (whether it is successful or not), the equilibrium payoff from attacking is $\mathbb{P}(\theta = 0|x = 1) + 0.5\mathbb{P}(\theta = 0.5|x = 1) - 0.4 = -0.025$; thus, speculators refrain from attacking when $x = 1$.
6. Finally, if $\theta = 1$, the size of the attack is $\mathbb{P}(x < 1|\theta = 1) = 0.5 < 0.7 = a(1)$,and the government defends the peg.

signal can lead to multiple equilibria. For example, if the government chooses to fully disclose the fundamentals by choosing the partition $\{\{0\},\{0.5\},\{1\}\}$, we are back to the common knowledge case and there are two equilibria.

Suppose that, in case of multiplicity, the government only cares about the worst equilibrium outcome. We claim that, in this case, the optimal partition is $\{\{0\},\{0.5,1\}\}$. The government thus sends two signals:

$$y = \begin{cases} y_l \quad , & \text{if } \theta = 0, \\ y_h \quad , & \text{if } \theta \in \{0.5, 1\}. \end{cases}$$

If the public signal is $y = y_l$, it becomes common knowledge that the true state is $\theta = 0$, which means that the currency peg will be abandoned and it is profitable to attack, regardless of the other speculators' behavior and the private signal. Hence, every speculator must attack in equilibrium after observing $y = y_l$. If $y = y_h$, it becomes common knowledge that it is never profitable to attack the currency peg when no one else attacks. This leads to an equilibrium in which speculators coordinate on not attacking if $y = y_h$.

It turns out the equilibrium is unique. Speculators follow the public signal and attack if and only if $y = y_l$, and the government abandons the peg if only if $\theta = 0$. The government is strictly better off by sending a public signal, since it eliminates attacks when $\theta = 1$. Without a public signal, when $\theta = 1$ half the speculators observe a private signal $x \leq 0.5$ and attack the peg, so the government has to pay a cost to defend the currency. Furthermore, in the worst equilibrium with full disclosure, the peg is abandoned if $\theta \in \{0, 0.5\}$. Thus, the optimal policy strictly dominates full disclosure (common knowledge) or no disclosure (uninformative public signal) for the government.

The remainder of paper shows that the results in this section still hold in a more general framework. In the full model, when the government only cares about the worst equilibrium outcome, it is without loss of generality to consider only two-signal structures. Public policy thus divides the fundamentals into two intervals: a lower interval, where the peg is abandoned and speculators coordinate on attacking; an upper interval, where the peg is maintained and no speculator attacks. The optimal policy involves maximizing the size of the upper interval, while keeping the equilibrium unique.

1.3. Model

1.3.1. Actions and payoffs

The model is similar to the one in Morris and Shin (1998), with the addition of a public signal. There is a currency peg in the economy and speculators have to decide whether to attack it or not. There is a continuum of speculators of measure one, who are indexed by i and uniformly distributed on $[0,1]$. The state of fundamentals in the economy is given by θ, which is uniformly distributed on $\Theta = [0,1]$. In the absence of government intervention, the exchange rate is a function $f(\cdot)$ of the state θ, where $f(\cdot)$ is continuous and strictly increasing. The exchange rate is initially pegged by the government at e^*, with $e^* \geq f(\theta)$ for all θ.

A speculator attacks the currency by selling short one unit of currency at a cost $t > 0$. If the speculator attacks and the peg is abandoned, his payoff is $e^* - f(\theta) - t$, whereas the payoff from attacking when the currency is defended is $-t$. If the speculator does not attack the currency, his payoff is zero.

The government derives a value $v > 0$ from maintaining the currency peg. There is a cost $c(\alpha, \theta)$ to defend the peg, where α is a mass of speculators who attack the currency. The cost c is continuous, strictly increasing in α and strictly decreasing in θ. Hence, the payoff from defending the peg is $v - c(\alpha, \theta)$, and the payoff from abandoning the peg is zero. The following assumptions are made:

- $c(0,0) > v$: the government abandons the peg if fundamentals are sufficiently weak, even if no one attacks;
- $c(1,1) > v$: the government abandons the peg if everyone attacks, even if fundamentals

are good;

- $e^* - f(1) - t < 0$: it is not profitable for speculators to attack the currency if fundamentals are good enough.

Denote by $\underline{\theta}$ the value of θ that solves $v = c(0, \theta)$. If $\theta \leq \underline{\theta}$, the government finds it optimal to abandon the peg regardless of the size of the attack. Denote by $\bar{\theta}$ the value of θ such that $e^* - f(\theta) - t = 0$. If $\theta > \bar{\theta}$, attacking is not profitable even if the peg is abandoned.

We assume that $\underline{\theta} < \bar{\theta}$.[5] When the state is common knowledge, we can divide Θ in three intervals, as it has been pointed out in the literature.[6] Following the terminology in Morris and Shin (1998):

- if $\theta \in [0, \underline{\theta}]$, the currency is *unstable*: the government always abandons the peg;
- if $\theta \in (\underline{\theta}, \bar{\theta})$, the currency is *ripe for attack*: a coordinated attack is profitable and, if there is coordination on not attacking, attacking is not profitable;
- if $\theta \in [\bar{\theta}, 1]$, the currency is *stable*: it is never profitable to attack the peg.

1.3.2. Timing and information

The game has three stages. In the first stage, before observing θ, the government commits to a disclosure policy, which is announced to the speculators. In the second stage, once θ is realized, a public signal y is sent according to the disclosure policy.[7] Speculators do not observe θ, just the public signal y and a private signal x. Given x and y, speculators simultaneously decide whether to attack the currency or not. In the last stage, the government observes θ and the size of the attack, and decides whether to defend the currency or

[5] This condition holds for a large v and a small t.

[6] See, for example, Obstfeld (1996) and Morris and Shin (1998).

[7] There are two interpretations for the disclosure policy. One is that the government commits to a disclosure rule, observes θ, and then sends the prescribed signal $y(\theta)$. Another interpretation is that the government commits to an information acquisition procedure and, if the state is θ, the government observes $y(\theta)$ and announces it. The latter is in line with the Bayesian persuasion literature (see, for example, Kamenica and Gentzkow (2011)). In this case, the government is not more informed than the speculators when the public signal is sent.

abandon the peg. The structure of the game is assumed to be common knowledge.

We denote a partition of the interval $[0,1]$ by $P = \{m_n\}_{n=0}^{N}$, where $0 = m_0 < m_1 < ... < m_N = 1$, and $N \in \mathbb{N}$.[8] The n-th interval of the partition P is denoted by y_n, with

$$y_1 = [0, m_1], y_2 = (m_1, m_2], ..., y_n = (m_{n-1}, m_n], ..., y_N = (m_{N-1}, 1].$$

When the public signal $y = y_n$ is sent, it becomes common knowledge that $\theta \in y_n$. When $N = 1$, the public signal is uninformative.

It is important to stress that, since the government commits to a choice of P before learning the true state θ, there is no *strategic learning*, i.e., the choice of P does not change the speculators' beliefs about what the government knows.[9] In Section 1.5, we show that commitment is essential for our results.

In addition to the public signal, speculator i observes a private signal x_i, where

$$x_i = \theta + \sigma\varepsilon_i,$$

with $\sigma > 0$. The idiosyncratic noise ε_i is drawn from a distribution with probability density function (pdf) $g(\cdot)$, and cumulative distribution function (cdf) $G(\cdot)$. Each ε_i is independently and identically distributed across agents and independent of θ. We assume that $\text{supp}(g) = [-\bar{\varepsilon}, \bar{\varepsilon}]$, $\bar{\varepsilon} > 0$, and that $g(\cdot)$ is differentiable on $(-\bar{\varepsilon}, \bar{\varepsilon})$. Define $\varepsilon = \sigma\bar{\varepsilon}$, and let $2\varepsilon < \min\{\underline{\theta}, 1 - \bar{\theta}\}$.

The derivative of $g(\cdot)$, $g'(\cdot)$, is assumed to be bounded and such that

$$\text{if } g'(\tilde{\varepsilon}) < 0, \text{ then } g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} \in (\tilde{\varepsilon}, \bar{\varepsilon}). \tag{1.1}$$

[8] In this presentation, we restrict the analysis to partitions with a finite number of intervals. The results still hold if the partitions can have a countable number of intervals.

[9] This is in contrast to Angeletos et al. (2006).

Since the common prior on θ is uniform on $[0,1]$, the posterior distribution of θ given private signal x and public signal y has probability density function $\phi_y(\theta|x)$, where

$$\phi_{y_n}(\theta|x) = \begin{cases} \frac{\frac{1}{\sigma}g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x-m_{n-1}}{\sigma}\right)-G\left(\frac{x-m_n}{\sigma}\right)}, & \text{if } \theta \in y_n \\ 0, & \text{otherwise} \end{cases}. \tag{1.2}$$

The derivation of $\phi_{y_n}(\theta|x)$ is presented in Appendix A.2.1.[10]

1.3.3. *Equilibrium*

We solve this game by backward induction. In the last stage, given an attack of size α and a state θ, the government optimally chooses to abandon the peg if and only if $c(\alpha, \theta) \geq v$. In the second stage, given a partition P, speculators observe the public signal and their own private signal. Anticipating the government's decision in the next stage, they simultaneously decide whether to attack the currency or not. In the first stage, the government chooses a partition P. The multiplicity in the second stage of the game poses a selection problem that we solve by assuming that the government only cares about the worst equilibrium outcome. Alternatively, we could assume that speculators play the equilibrium strategy that maximizes their own payoff (or, equivalently, the one that minimizes the government's payoff).

More formally, suppose the government chooses a partition $P = \{m_n\}_{n=0}^N$. Let $p_n = \mathbb{P}(\theta \in y_n)$ be the probability that θ lies in the interval y_n of the partition.[11] In addition, consider the subgame that follows the disclosure of $y = y_n$. Denote V_n the infimum of all government's equilibrium payoffs when $y = y_n$.[12] We let $V(P) = \sum_{n=1}^N p_n V_n$. The government's problem is to choose P to maximize $V(P)$.

[10]There is a finite number of pairs (x, y) that fully reveal θ: when $y = y_n$ and $x = m_n + \varepsilon$, we have $\mathbb{P}(\theta = m_n | y = y_n, x = m_n + \varepsilon) = 1$; likewise, when $y = y_1$ and $x = -\varepsilon$, then $\mathbb{P}(\theta = 0 | y = y_1, x = -\varepsilon) = 1$. For all other pairs (x, y), the conditional density of θ is given by (1.2).

[11]Since we assume that θ is uniformly distributed on $[0,1]$, we have $p_n = m_n - m_{n-1}$.

[12]Such infimum always exists as the government always has the option to abandon the peg, so the equilibrium payoff is bounded below by 0.

The problem of the government in the last stage is simple. For each θ, let $a(\theta)$ be the solution to $v = c(a, \theta)$. This function represents the critical mass of speculators that have to attack the currency in order to induce the government to abandon the peg. Note that, given our assumptions on $c(\cdot, \cdot)$, we have that $a(\cdot)$ is continuous, $a(\theta) = 0$ for $\theta \leq \underline{\theta}$, and $a(\cdot)$ is strictly increasing for $\theta > \underline{\theta}$.

For a given profile of strategies for the speculators, the measure of speculators who attack the currency given a pair of signals (x, y) is denoted by $\pi(x, y)$. If the state is θ, the proportion of speculators who attack the currency is given by

$$s(\theta, \pi) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x, y_{n(\theta)}) \frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) dx. \tag{1.3}$$

where $y_{n(\theta)}$ is the public signal sent according to P when the state is θ. The government maintains the peg when

$$s(\theta, \pi) < a(\theta). \tag{1.4}$$

Thus, the event where there is a regime change is given by

$$A(\pi) = \{\theta : s(\theta, \pi) \geq a(\theta)\}. \tag{1.5}$$

The payoff to a speculator from attacking the currency at state θ, when the aggregate strategy is π, is given by

$$h(\theta, \pi) = \begin{cases} e^* - f(\theta) - t, & \text{if } \theta \in A(\pi) \\ -t, & \text{if } \theta \notin A(\pi) \end{cases}. \tag{1.6}$$

The expected payoff from attacking the currency given a pair of signals (x, y_n) is given by[13]

$$u_{y_n}(x,\pi) = \int_{[x-\varepsilon,x+\varepsilon]\cap y_n} h(\theta,\pi)\phi_{y_n}(\theta|x)d\theta \tag{1.8}$$
$$= \int_{[x-\varepsilon,x+\varepsilon]\cap y_n\cap A(\pi)} [e^* - f(\theta)]\phi_{y_n}(\theta|x)d\theta - t.$$

In equilibrium, $\pi(x, y) = 1$ if $u_y(x, \pi) > 0$, and $\pi(x, y) = 0$ if $u_y(x, \pi) < 0$.

1.3.4. Equilibrium properties

We now present some auxiliary results, which are similar to the ones in Morris and Shin (1998). The first result shows that, if other speculators are more likely to attack the currency peg for every private signal x, then the expected payoff from attacking increases.

Lemma 1 *For a given public signal y, if $\pi(x, y) \geq \pi'(x, y)$ for all x, then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x.*

Proof: See Appendix A.2.2. □

For $k \in [-\varepsilon, 1 + \varepsilon]$, let the indicator function I_k be defined as

$$I_k(x) = \begin{cases} 1, & \text{if } x < k \\ 0, & \text{if } x \geq k \end{cases}. \tag{1.9}$$

When aggregate short sales are given by I_k (in particular, short sales will not depend on the public signal y), the proportion of speculators who attack the currency at state θ is given

[13] Equation (1.7) holds for all but a finite number of pairs (x, y), as described in footnote 10. If $y = y_n$ and $x = m_n + \varepsilon$,

$$u_{y_n}(m_n + \varepsilon, \pi) = [e^* - f(m_n)]\mathcal{I}(m_n) - t, \tag{1.7}$$

where $\mathcal{I}(\theta)$ is an indicator function that equals 1 if the peg is abandoned at state θ. Similarly,

$$u_{y_1}(-\varepsilon, \pi) = e^* - f(0) - t,$$

since the peg is always abandoned when $\theta = 0$.

by

$$s(\theta, I_k) = G\left(\frac{k-\theta}{\sigma}\right). \tag{1.10}$$

Note that $s(\theta, I_k)$ is strictly decreasing in θ for $\theta \in (k-\varepsilon, k+\varepsilon)$, and constant otherwise.

We denote by θ_k the largest value of θ at which the government finds it optimal to abandon the currency peg when the speculators' aggregate short sales are given by I_k. As in Morris and Shin (1998), let $\psi(k) = \min\{\theta_k - k, \varepsilon\}$. Appendix A.2.3 provides a derivation of θ_k and ψ. The threshold θ_k is increasing in k, and the government finds it optimal to abandon the peg for all $\theta \leq \theta_k$. The function $\psi(\cdot)$ is continuous, $\psi(k) = \varepsilon$ for $k \leq \underline{\theta} - \varepsilon$, $\psi(1+\varepsilon) = -\varepsilon$, and $\psi(\cdot)$ is strictly decreasing for $k \in (\underline{\theta} - \varepsilon, 1+\varepsilon]$.

Let X_y denote the set of private signals that can be received by the speculators when the public signal is y. Then $X_{y_1} = [-\varepsilon, m_1 + \varepsilon]$ and, for $n > 1$, $X_{y_n} = (m_{n-1} - \varepsilon, m_n + \varepsilon]$.

Since the currency peg is abandoned if and only if $\theta \in [0, k + \psi(k)]$, the payoff function $u_{y_n}(k, I_k)$ is given by

$$u_{y_n}(k, I_k) = \int_{[k-\varepsilon, k+\psi(k)] \cap y_n} [e^* - f(\theta)] \phi_{y_n}(\theta | k) d\theta - t, \tag{1.11}$$

for all $k \in X_{y_n}$.[14]

Lemma 2 *For a given public signal y, $u_y(k, I_k)$ is continuous in k, for all possible private signals $k \in X_y$.*

[14] As in footnote 13, equation (1.11) holds for all but a finite number of (x, y) that fully reveal θ. The reader can check that

$$lim_{k \to m_n + \varepsilon} u_{y_n}(k, I_k) = u_{y_n}(m_n + \varepsilon, I_{m_n + \varepsilon}), \forall n,$$
$$lim_{k \to -\varepsilon} u_{y_1}(k, I_k) = u_{y_1}(-\varepsilon, I_{-\varepsilon}),$$

and that for a fixed k,

$$lim_{x \to m_n + \varepsilon} u_{y_n}(x, I_k) = u_{y_n}(m_n + \varepsilon, I_k), \forall n,$$
$$lim_{x \to -\varepsilon} u_{y_1}(x, I_k) = u_{y_1}(-\varepsilon, I_k).$$

For the sake of brevity, in the remainder of the paper we omit these finite number of cases. The limits above guarantee that our results still hold.

Proof: See Appendix A.2.4. □

Let $u(k, I_k)$ be the payoff function when there is no public signal. Then

$$u(k, I_k) = \int_a^b [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x}{\sigma}\right) - G\left(\frac{x-1}{\sigma}\right)} d\theta - t, \tag{1.12}$$

where $a = \max\{k - \varepsilon, 0\}$, and $b = k + \psi(k)$. Note that the payoff function is continuous in k. The following lemma shows that it is also strictly decreasing in k.

Lemma 3 *For $k \in (\varepsilon, 1-\varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k.*

Proof: See Appendix A.2.5. □

1.4. Optimal signal structure

This section presents the results of the model with commitment when, in case of multiplicity after a partition choice, the government only cares about the worst equilibrium outcome.[15] First, we show that there is no loss of generality in considering partitions with at most two intervals. Then, we prove that it is not optimal for the government to chose a one-interval partition and send the uninformative signal. Finally, the optimal partition is characterized.

1.4.1. No loss of generality in two-interval partitions

Let $\Phi_y(\theta|x)$ denote the cumulative distribution function of θ conditional on private signal x and public signal y. To find the optimal partition, the following assumption is made.

Assumption 1 *Let the public signal be y. For any pair of private signals x_1 and x_2, with $x_1 < x_2$, $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for all θ.*

This assumption means that the distribution of θ conditional on y and x_2 first-order stochastically dominates the distribution of θ conditional on y and x_1. In Appendix A.2.6, it is shown that Assumption 1 is satisfied, for example, if the idiosyncratic noise on $[-\bar{\varepsilon}, \bar{\varepsilon}]$ fol-

[15]When there is no ambiguity, we say equilibrium when we mean the equilibrium of the subgame that follows the choice of P.

lows a concave or a truncated normal distribution. Assumption 1 leads to the following lemma.

Lemma 4 *Suppose that Assumption 1 is satisfied. When the aggregate strategy is given by I_k, the payoff from attacking the currency, $u_y(x, I_k)$, is decreasing in the private signal x.*

Proof: Suppose that the aggregate strategy is given by I_k. Let $\mathcal{I}(\theta)$ be an indicator function that equals 1 if the currency peg is abandoned when the state is θ. Since, by assumption, speculators follow a cutoff rule, $\mathcal{I}(\theta)$ is weakly decreasing in θ.[16] Define

$$U(\theta) = [f(\theta) - e^*]\mathcal{I}(\theta),$$

which is negative and increasing. Consider a public signal y and a pair of private signals x_1 and x_2, with $x_1 < x_2$. Then

$$\int_0^1 U(\theta) d\Phi_y(\theta|x_2) \geq \int_0^1 U(\theta) d\Phi_y(\theta|x_1),$$

where the inequality comes from Assumption 1 and the fact that U is increasing. Hence

$$\begin{aligned} u_y(x_1, I_k) &= -\int_0^1 U(\theta) d\Phi_y(\theta|x_1) - t \\ &\geq -\int_0^1 U(\theta) d\Phi_y(\theta|x_2) - t \\ &= u_y(x_2, I_k), \end{aligned}$$

which completes the proof. □

The following two lemmas are needed for the main results. The first one shows a sufficient condition for a cutoff strategy for the speculators to exist in equilibrium, while the second lemma characterizes the speculators' equilibrium strategy that minimizes the government's payoff for a given public signal y.

[16] $\mathcal{I}(\theta) = 1$, if $\theta \leq \theta_k$; and $\mathcal{I}(\theta) = 0$, if $\theta > \theta_k$.

Lemma 5 *Let the public signal be y, and suppose that Assumption 1 is satisfied. If k solves $u_y(k, I_k) = 0$, then there is an equilibrium where the aggregate short sales after y is observed are given by I_k.*

Proof: Fix y. Suppose that k solves $u_y(k, I_k) = 0$, and that the aggregate short sales are given by I_k. If a speculator receives a signal $x < k$, his payoff from attacking the currency is given by

$$u_y(x, I_k) \geq u_y(k, I_k) = 0,$$

where the inequality comes from Lemma 4. Hence the payoff from attacking is (weakly) larger than the payoff from not attacking. Similarly, if $x \geq k$, the payoff from attacking the currency is

$$u_y(x, I_k) \leq u_y(k, I_k) = 0,$$

therefore not attacking yields a (weakly) larger payoff than attacking. Both statements imply that following a cutoff rule I_k is optimal for the speculator, given that all other speculators are using the same rule. This means that there exists an equilibrium in which I_k is the aggregate selling strategy. □

Lemma 6 *Suppose that Assumption 1 is satisfied. For a given public signal y,*

i. if $u_y(k, I_k) < 0$ for all $k \in X_y$, then, in any equilibrium, $\pi(x, y) = 0$ for all $x \in X_y$.

ii. if $u_y(k', I_{k'}) \geq 0$ for some $k' \in X_y$, then, in the worst equilibrium for the government, speculators use the cutoff rule I_k after observing y, where $k = \sup\{k' \in X_{y_n} : u_{y_n}(k', I_{k'}) \geq 0\}$.

Proof:

i. Suppose that $u_y(k, I_k) < 0$ for all $k \in X_y$. Let π be any equilibrium strategy, and suppose by way of contradiction that there is $x' \in X_y$ such that $\pi(x', y) > 0$. If this is true, then the set $\{x \in X_y : \pi(x,y) > 0\}$ is non-empty and we can define $\bar{x}_y$ as

$$\bar{x}_y = \sup\{x \in X_y : \pi(x,y) > 0\}.$$

Note that $\bar{x}_y \in X_y$ because X_y is right-closed. Also note that, if π is an equilibrium strategy, then for any $\tilde{x} \in \{x \in X_y : \pi(x,y) > 0\}$, it has to be true that $u_y(\tilde{x},\pi) \geq 0$. By the continuity of u_y in the private signal, $u_y(\bar{x}_y,\pi) \geq 0$. From Lemma 1,

$$u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y,\pi) \geq 0$$

Thus, $u_y(\bar{x}_y, I_{\bar{x}_y}) \geq 0$, which contradicts the assumption that $u_y(k, I_k) < 0$ for all $k \in X_y$.

ii. If $u(k, I_k) > 0$, by continuity (Lemma 2), it has to be true that k is the right bound of the interval X_y and, by the decreasing property of u_y in x (Lemma 4), I_k is an equilibrium strategy. If $u(k, I_k) = 0$, then we know that I_k is an equilibrium strategy (Lemma 5). Using the same arguments from part *i.*, assume by way of contradiction that there is an equilibrium with $\pi(x',y) > 0$ for some $x' > k$. Let $\bar{x}_y = \sup\{x \in X_y : \pi(x,y) > 0\} \in X_y$. By Lemma 1, $u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y,\pi) \geq 0$, which contradicts the assumption that k is the supremum of the set $\{k' \in X_y : u_y(k', I_{k'}) \geq 0\}$.

□

Recall that $\bar{\theta}$ defines the threshold θ at which investors' payoff from a successful attack is 0. The following lemma characterizes the worst equilibrium outcome for the government given a choice of P.

Lemma 7 *Suppose that Assumption 1 is satisfied and consider an arbitrary partition* $P = \{m_n\}_{n=0}^N$. *Given* P, *the equilibrium that minimizes the government's payoff involves the following:*

i. for all n *such that* $m_n \leq \bar{\theta}$, *speculators always attack the currency if* $y = y_n$;

ii. for all n such that $m_{n-1} \geq \bar{\theta}$, speculators never attack the currency if $y = y_n$;

iii. for all n such that $m_{n-1} < \bar{\theta}$ and $m_n > \bar{\theta}$,[17] speculators never attack if $u_{y_n}(k, I_k) < 0$ for all $k \in X_{y_n}$. Otherwise, speculators follow I_{k_n} after observing y_n, where $k_n = \sup\{k \in X_{y_n} : u_{y_n}(k, I_k) \geq 0\}$.

Proof: i. Let n be such that $m_n \leq \bar{\theta}$. If speculators always attack the currency after observing y_n, then the government abandons the peg (because $c(1,1) > v$) and the speculators have a positive payoff (by the definition of $\bar{\theta}$). Hence always attacking after observing y_n is an equilibrium strategy for the speculators, and no other strategy can yield a lower payoff for the government when $\theta \in y_n$.

ii. Let n be such that $m_{n-1} \geq \bar{\theta}$. If a speculator attacks after observing y_n, his expected payoff is strictly negative. Hence there is no equilibrium where speculators attack when $\theta \in y_n$.

iii. Follows immediately from Lemma 6. □

Lemma 7 provides the intuition as to why there is no loss of generality in considering only two-interval partitions. If there are several n such that $m_n \leq \bar{\theta}$, then the government could group all these y_n. Likewise, if there are several n such that $m_{n-1} \geq \bar{\theta}$, the government can group these y_n. This rules out any partition P with four or more intervals.

Now consider a partition P with three intervals, that is, $P = \{0, m_1, m_2, 1\}$, where $m_1 < \bar{\theta} < m_2$. If the government maintains the peg for all $\theta \in (m_1, m_2]$, then the government could have chosen the partition $P' = \{0, m_1, 1\}$. If the government abandons the peg for all $\theta \in (m_1, m_2]$, then the government could have chosen the partition $P' = \{0, m_2, 1\}$. If the government abandons the peg for some but not all $\theta \in (m_1, m_2]$, then, by Lemma 7, speculators use a cutoff rule when $y = (m_1, m_2]$. This cutoff strategy generates a threshold $\theta' \in y_2$, such that the government abandons the peg if $\theta \in (m_1, \theta']$ and maintains the peg

[17] There is at most one such n.

if $\theta \in (\theta', m_2]$. But if this is the case, then the government could have chosen the partition $P' = \{0, \theta', 1\}$. We use Lemma 15 (in the appendix A.2.7) to formalize this result, which is presented in Theorem 1.

Theorem 1 *Suppose that Assumption 1 is satisfied. Then, for any partition* $P = \{m_n\}_{n=0}^{N}$ *with* $N > 2$*, there exists* $P' = \{m'_n\}_{n=0}^{N'}$ *with* $N' = 2$*, such that* $V(P') \geq V(P)$*.*

Proof: Given Lemma 7, the only non trivial result left to show is that, for any $P = \{0, m_1, m_2, 1\}$, with $m_1 < \bar{\theta} < m_2$, there is a $P' = \{0, m', 1\}$ such that $V(P') \geq V(P)$.

- *Case 1: the government maintains the peg for all θ in y_2.* Consider the alternative partition $P' = \{0, m_1, 1\}$. The government cannot be worse off if $\theta \leq m_1$.

  We know from Lemma 7 that $u_{(m_1,m_2]}(k, I_k) < 0$ for any $k \in X_{(m_1,m_2]}$. Since $m_2 > \bar{\theta}$, we also know that $u_{(m_2,1]}(k, I_k) < 0$ for any $k \in X_{(m_2,1]}$. From Lemma 15,

  $$u_{(m_1,1]}(k, I_k) \leq u_{(m_1,m_2]}(k, I_k) < 0, \quad \text{for any } k \in (m_1 - \varepsilon, m_2 + \varepsilon],$$

  and

  $$u_{(m_1,1]}(k, I_k) = u_{(m_2,1]}(k, I_k) < 0, \quad \text{for any } k \in (m_2 + \varepsilon, 1 + \varepsilon].$$

  The inequalities imply that $u_{(m_1,1]}(k, I_k) < 0$ for $k \in X_{(m_1,1]}$. From Lemma 6, no one attacks if $\theta > m_1$. Thus, $V(P') \geq V(P)$.

- *Case 2: the government abandons the peg for all θ in y_2.* Consider the partition $P' = \{0, m_2, 1\}$. The government is not worse off if $\theta \leq m_2$. If $\theta > m_2$, speculators observe the public signal $(m_2, 1]$, and since $m_2 > \bar{\theta}$, no one attacks. Thus, $V(P') \geq V(P)$.

- *Case 3: the government abandons the peg at some but not all θ in y_2.* From Lemma 6, speculators use a cutoff rule I_{k_2}.

  From Lemma 7, speculators follow a cutoff rule I_{k_2} after observing y_2, where $k_2 = \sup\{k \in X_{y_2} : u_{y_2}(k, I_k) = 0\}$. Given the speculators' strategy, there exists $\theta_{k_2} \in$

$(m_1, m_2]$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_2}$. From Lemma 15, increasing m_1 would never increase the cutoff k, and it would never increase the threshold θ_k.

This implies that, with the partition $\tilde{P}' = \{0, \theta_{k_2}, m_2, 1\}$, no one attacks if $\theta \in (\theta_{k_2}, m_2]$. From Case 2, when the partition is $P' = \{0, \theta_{k_2}, 1\}$, there is no attack if $\theta \in (\theta_{k_2}, m_2]$. By changing the partition from P to P', the government no longer has to pay a cost to defend the currency on $(\theta_{k_2}, \theta_{k_2} + \varepsilon)$, therefore $V(P') > V(P)$.

□

For the remainder of the paper, we denote the two element partition $P = \{0, m, 1\}$ by P^m.

1.4.2. No disclosure is not optimal

This subsection shows that it is not optimal for the government to send the uninformative public signal, i.e., to set $N = 1$. The result is obtained by proving that there exist partition choices with $N = 2$ that strictly dominate the uninformative partition with $N = 1$. When $N = 2$, the government's problem is equivalent to a choice of $m \in [0, 1]$ such that speculators will learn whether $\theta \leq m$ or $\theta > m$. Given the choice of m, they observe the public signal $y \in \{y_l, y_h\}$, drawn as follows:[18]

$$y = \begin{cases} y_l, & \text{if } \theta \in [0, m] \\ y_h, & \text{if } \theta \in (m, 1] \end{cases} . \tag{1.13}$$

In the model without a public signal, which has the same outcome as the case $m = 1$, it is known from Morris and Shin (1998) that the equilibrium is unique. In that equilibrium, speculators follow a cutoff rule and attack the currency if and only if their private sigal is below x^*, where x^* solves $u(x^*, I_{x^*}) = 0$. The currency peg is thus abandoned if and only

[18]To distinguish the case where the government is restricted to $N \leq 2$ from the general case, we change the notation: we use m instead of m_1; y_l and y_h instead of y_1 and y_2.

if $\theta \leq \theta^*$, where θ^* makes the government indifferent between defending the peg or not.[19]

The next lemma shows that, for any choice of $m < 1$, there is always an equilibrium where government and speculators coordinate on the public signal for at least one realization of y.

Lemma 8 *Consider the subgame that follows the choice of $m < 1$ by the government. If $m \leq \bar{\theta}$, there exists an equilibrium where the government abandons the peg when $\theta \in [0, m]$, and the speculators attack the currency after observing $y = y_l$. If $m \geq \underline{\theta}$, there exists an equilibrium where the government defends the peg if $\theta \in (m, 1]$, and there is no attack following the signal y_h.*

Proof: Let $m \leq \bar{\theta}$ and suppose that all speculators attack the currency after observing $y = y_l$. Given the speculators' aggregate strategy, the government abandons the currency peg if $\theta \in [0, m]$, and it is indeed optimal for each speculator to attack if $y = y_l$. Now let $m \geq \underline{\theta}$ and suppose that no speculator attacks the currency after observing $y = y_h$. Given the speculators' strategy, the government defends the peg if $\theta \in (m, 1]$, and therefore it is indeed optimal for each speculator not to attack if $y = y_h$. $\square$

For any choice of $m \in [\underline{\theta}, \bar{\theta}]$, there exist an equilibrium where speculators follow the public signal: they coordinate on attacking if $y = y_l$, and they refrain from attacking if $y = y_h$. In this equilibrium, the currency peg is abandoned if $\theta \leq m$. This result is presented in Corollary 1 below.

Corollary 1 *For all $m \in [\underline{\theta}, \bar{\theta}]$, there exists an equilibrium where the currency peg is abandoned if and only if $\theta \in [0, m]$, and speculators attack the currency if and only if $\theta \in [0, m]$.*

The following theorem compares the equilibrium outcomes for a given choice of m with the unique equilibrium outcome in the absence of a public signal.

Theorem 2 *Fix m. If $m = \theta^*$, there is a unique equilibrium, in which speculators follow the public signal. If $m \neq \theta^*$, the equilibrium may not be unique. There are bounds $\underline{x}^* \geq x^*$*

[19] That is, θ^* solves $s(\theta, I_{x^*}) = a(\theta)$.

and $\bar{x}^* \leq x^*$ *such that, in any equilibrium,* $\pi(x, y_l) \geq I_{\underline{x}^*}(x)$ *and* $\pi(x, y_h) \leq I_{\bar{x}^*}(x)$ *for all* x*. The equilibria are as follows:*

i. if $m < \theta^*$*: speculators always attack the currency and the peg is abandoned if* $y = y_l$*; moreover, if* $m \in (x^* - \varepsilon, \theta^*)$*, then* $\bar{x}^* < x^*$*;*

ii. if $m > \theta^*$*: the currency is not attacked and the peg is defended if* $y = y_h$*; moreover, if* $m \in (\theta^*, x^* + \varepsilon)$*, then* $\underline{x}^* > x^*$*.*

Proof: See Appendix A.2.8. □

Part *i.* of Theorem 2 states that, when the government chooses $m < \theta^*$ and the public signal is $y = y_h$, the set of private signals that induce attack is contained in the set of private signals that would induce attack in the absence of a public signal. Thus, for any $\theta \in y_h$, the size of the attack does not increase. Moreover, if Assumption 1 holds, we use Lemma 7 to conclude that, in the worst equilibrium for the government, the cutoff used when $y = y_h$ is below the cutoff when there is no public signal. Conversely, part *ii.* and Assumption 1 imply that, when $m > \theta^*$, speculators will use a higher cutoff when $y = y_l$. This can be seen in Figure 1, which is constructed from the numerical example in Appendix A.

If the government chooses $m = \theta^*$, the equilibrium is unique and the currency peg is abandoned if and only if $\theta \leq \theta^*$, as in the equilibrium of the game without a public signal. However, no speculator attacks the currency when $\theta > \theta^*$, whereas without the public signal some speculators would still attack the currency for some $\theta > \theta^*$, increasing the cost of maintaining the peg. Thus, the government is strictly better off by the introduction of the public signal. Note that the speculators are also strictly better off now that all of them attack when the currency peg is abandoned, and no one attacks when peg is maintained.

Table 1 summarizes the results in Theorem 2.[20]

		Probability of Devaluation	Equilibrium payoff
	$m \le x^* - \varepsilon$	weakly lower	weakly higher
$x^* - \varepsilon <$	$m \le \theta^*$	strictly lower	strictly higher
$\theta^* <$	$m < x^* + \varepsilon$	strictly higher	effect is ambiguous
	$m \ge x^* + \varepsilon$	weakly higher	weakly lower

Table 1: Given m, the second column compares the equilibrium probability of currency devaluation with the case without a public signal. The last column compares the government's possible equilibrium payoffs with the unique equilibrium payoff in the game without a public signal.

Note that any choice $m > \theta^*$ is strictly dominated by $m = \theta^*$. Compared to the unique equilibrium with $m = \theta^*$, any equilibrium with $m > \theta^*$ features a strictly higher probability of devaluation and, for all θ, there is a weakly larger mass of speculators attacking the currency. This leads to the following corollary.

Corollary 2 *The choice of any $m > \theta^*$ is strictly dominated by $m = \theta^*$.*

Corollary 2 implies that the choice of $m = 1$ is strictly dominated by choosing $N = 2$ and $m = \theta^*$. Thus sending an uninformative public signal is not optimal for the government.

1.4.3. Characterization of the optimal signal structure

We are now ready to find the optimal partition for the government. Define M as

$$M = \{m : \text{ there is no attack in any equilibrium after } y_h\}.$$

Note that $M \neq \emptyset$ because $\theta^* \in M$. Define $\underline{m}$ as

$$\underline{m} = \inf M.$$

Figure 2 from Appendix A gives the intuition about how the optimal partition should be. Any choice of $m > \theta^*$ is strictly dominated by $m = \theta^*$, which leads to a unique equilibrium,

[20] If $m > \theta^*$, the currency is not attacked when $y = y_h$. For m close enough to θ^*, the government's payoff can be higher with the public signal if the increase in the probability of devaluation when $y = y_l$ is offset by a lower cost of defending the currency when $y = y_h$.

with no attacks when $\theta \in (m, 1]$. Starting from $m = \theta^*$, as m decreases, the government is strictly better off as long as the equilibrium is still unique. Decreasing m will increase the range of fundamentals that lead to no currency attacks. However, there is a discontinuity in the government's payoff at $\underline{m}$. Decreasing m even further to the region where it leads to multiple equilibria makes the government strictly worse off. Thus, the government wants decrease m up to the limit where the equilibrium is still unique, $\underline{m}$. This result is formalized in the following theorem.

Theorem 3 *Suppose that Assumption 1 is satisfied. For every partition P, $V(P) \leq \overline{V}$, where*

$$\overline{V} = \lim_{m \downarrow \underline{m}} V(P^m).$$

Then

i. *if $\underline{m} \in M$, the government's equilibrium payoff is $\overline{V}$. In equilibrium, when $\theta > \underline{m}$, there are no attacks and the peg is maintained; and when $\theta \leq \underline{m}$, every speculator attacks the currency and the peg is abandoned. The government can achieve the payoff $\overline{V}$ with the two-interval partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$.*

ii. *if $\underline{m} \notin M$, no equilibrium exists. However, the government can achieve a payoff arbitrarily close to $\overline{V}$.*

Proof: See Appendix A.2.10. □

The optimal policy involves setting m as low as possible, up to the limit where not attacking is the unique equilibrium action for speculators on $(m, 1]$. Note that for any m close enough to $\underline{m}$[21] any disclosure policy with $y_N = (m, 1]$ yields the same payoff for the government, regardless of the signal structure when $\theta \in [0, m]$. It is still true that when $\theta > m$, there is no attack and the peg is maintained; and when $\theta \leq m$, every speculator attacks the currency and the peg is abandoned. Thus the government could be arbitrarily precise when the state

[21] That is, $m \in (\underline{m}, \bar{\theta})$.

of fundamentals is very bad, but when the state is "not too bad" the government needs to be vague. This vagueness is used by the government to make the speculators uncertain about whether the state is intermediate (where a coordinated attack is profitable) or good (when attacking is never profitable), preventing them from attacking.

1.4.4. Vagueness

We conclude this section by showing that, even if the government could fully disclose the state, it would not be optimal to do so.

In Lemma 18 (Appendix A.2.9), we show that that $\underline{m} < \bar{\theta}$. Since $\underline{m} < \bar{\theta}$, there exists $m \in M \cap (\underline{m}, \bar{\theta})$ such that the partition $P^m = \{0, m, 1\}$ is strictly preferred to full disclosure. If the state is fully revealed, in the worst equilibrium for the government, speculators coordinate on attacking whenever $\theta < \bar{\theta}$. With the partition P^m, the government eliminates currency attacks between $(m, \bar{\theta})$. This leads to Proposition 1.

Proposition 1 *Full disclosure of the state is not an optimal policy for the government.*

1.5. No commitment

In this section, we drop the assumption that the government can commit to a disclosure policy. Here the government chooses the public signal after observing the true state θ. For simplicity, the government's strategy in the last stage of the game is taken as given.

The game between government and speculators becomes a signaling game, where θ can be interpreted as the government's type. A strategy for the government is a function $y : \Theta \to \Theta^2$ such that when the state is θ, the public signal is $y(\theta) = [\underline{y}(\theta), \overline{y}(\theta)]$ and speculators learn that $\theta \in [\underline{y}, \overline{y}]$.[22] As before, we require that the government must make truthful announcements, that is, $\underline{y}(\theta) \leq \theta \leq \overline{y}(\theta)$ for all θ. Note that, if the government is not restricted to truthful announcements, there exists an equilibrium in which the speculators ignore the public signal. In this case, the equilibrium outcome is the same as the one in the

[22]The restriction to closed intervals is made only for simplicity.

game without a public signal. (And, possibly, there are even worse equilibria.)

A strategy for speculators is a function that gives, for every private signal x and every public signal, the corresponding action to be taken (attack or not). As before, let $\pi(x, y)$ be the aggregate selling strategy. The equilibrium concept in this section is the Perfect Bayesian Equilibrium (PBE) with symmetric strategies for the speculators.

Definition. The strategy profile (y, π) is a PBE if

1. for all $\theta \in [0, 1]$, $y(\theta)$ maximizes the government's payoff given π;
2. for each possible signal y, there exist beliefs $\mu_{x,y}$ about θ such that $\pi(x, y)$ maximizes the speculator's expected payoff given $\mu_{x,y}$, the aggregate strategy π, and signals x and y;
3. for each signal y such that $\int_{\{\tilde{\theta}:y(\tilde{\theta})=y\}} 1d\tilde{\theta} > 0$,

$$\mu_{x,y}(\theta) = \begin{cases} \frac{\frac{1}{\sigma}g(\frac{x-\theta}{\sigma})}{\int_{\{\tilde{\theta}:y(\tilde{\theta})=y\}} 1d\tilde{\theta}}, & \text{if } y(\theta) = y \\ 0, & \text{otherwise} \end{cases} . \tag{1.14}$$

4. for each signal y such that $\int_{\{\tilde{\theta}:y(\tilde{\theta})=y\}} 1d\tilde{\theta} = 0$

$$\text{support}(\mu_{x,y}(\theta)) \subset [x - \varepsilon, x + \varepsilon] \cap y \tag{1.15}$$

Consider the following profile of (y, π, μ):

$$
\begin{aligned}
y(\theta) =& \{\theta\}, \quad \forall \theta, \\
\mu_{x,y}(\theta) =& \begin{cases} 1, & \text{if } \theta = \max\{x - \varepsilon, \underline{y}\} \\ 0, & \text{otherwise} \end{cases} \\
\pi(x, y) =& \begin{cases} 1, & \text{if } \max\{x - \varepsilon, \underline{y}\} \leq \bar{\theta} \\ 0, & \text{otherwise} \end{cases} .
\end{aligned}
$$

In equilibrium, the public signal reveals the true state of the fundamentals, and speculators attack if and only if $\theta < \bar{\theta}$. Since $\theta^* < \bar{\theta}$, the government is ex ante strictly worse off compared to the unique equilibrium of the game without a public signal. Furthermore, for all types θ the equilibrium payoff is weakly smaller.

To see that (y, π) is in fact an equilibrium with beliefs μ, first consider speculator i's problem. If $\underline{y} = \overline{y} = \theta$, given that speculators follow π, it is only profitable for i to attack if $\underline{y} < \bar{\theta}$, which means that π is optimal on path. Now consider off path signals where $\underline{y} < \overline{y}$. When $\underline{y} \geq \bar{\theta}$, the speculators know that $\theta \geq \bar{\theta}$ and attacking is indeed not profitable. If $\underline{y} < \bar{\theta}$ and speculator i receives a private signal $x_i \leq \bar{\theta} + \varepsilon$, he believes that $\theta = \max\{x - \varepsilon, \underline{y}\} < \bar{\theta}$. The speculator also believes that everyone else received a private signal below $\bar{\theta} + \varepsilon$, and if aggregate sales are given by π, the speculator believes that other speculators will attack. Hence, attacking is profitable given μ, π, x_i, and y. Finally, if $\underline{y} < \bar{\theta}$ and $x_i > \bar{\theta} + \varepsilon$, the speculator knows that $\theta \geq \bar{\theta}$, and it is not profitable to attack. Thus, following π is optimal when $\underline{y} < \bar{\theta}$ and $x_i > \bar{\theta} + \varepsilon$.

Now we show that strategy y is optimal for the government given π and μ. Suppose that the government has a profitable deviation from y for some type $\theta' \in [0, 1]$.[23] Since there is no attack for $\theta \geq \bar{\theta}$, there can only be a profitable deviation if $\theta' < \bar{\theta}$. According to μ, speculators believe that $\theta < \bar{\theta}$ and attack. Thus, there is no profitable deviation from y.

[23] A deviation here is a signal signal $y = [\underline{y}, \overline{y}]$, with $\underline{y} < \overline{y}$ and $\theta' \in [\underline{y}, \overline{y}]$.

The PBE above passes the intuitive criterion of Cho and Kreps (1987). Note that only types in $[0, \bar{\theta})$ could benefit from a deviation. However, if the speculators know that $\theta < \bar{\theta}$, they can coordinate on attacking the currency peg, thus a deviation would not be profitable.

This example shows that commitment is essential for the government to benefit from a public signal. When only truthful announcements are allowed, the speculators can exploit the fact that the government wants to reveal its type for $\theta \geq \bar{\theta}$. In this case, speculators are able to coordinate on attacking the currency peg whenever $\theta < \bar{\theta}$. If the government is allowed to lie, the speculators can simply ignore the public signal. The results are summarized in the following proposition.

Proposition 2 *Suppose that the government only cares about its lowest equilibrium payoff. If the government cannot commit to a disclosure policy, then it does not benefit from sending a public signal. Furthermore, when only truthful announcements are allowed, the government is strictly worse off with the introduction of a public signal.*

1.6. Conclusion

This paper analyzes a model of currency attacks in which the government sends a credible public signal about the fundamentals of the economy. The government can partition the space of fundamentals and reveal in which interval the unknown state lies. The introduction of a public signal generates partial common knowledge about the fundamentals and it can lead to multiple equilibria. We find informative policies that strictly dominate no disclosure even if multiplicity arises. We also derive the optimal policy by assuming that the government only cares about the worst equilibrium outcome of each policy.

We find that sending very precise public signals can be harmful to the government. In fact, for any signal structure, there exists a two-signal policy that is preferred by the government. The optimal disclosure policy thus partitions the space of fundamentals into two intervals. In the lower interval, speculators coordinate on attacking the currency and the peg is abandoned; in the higher interval, no speculator attacks and the peg is maintained.

The government is deliberately vague in order to induce speculators not to attack the currency in the higher interval. After the public signal reveals that the state is in the higher interval, speculators are not sure whether a coordinated attack is profitable or not, thus they refrain from attacking. If the government had chosen a finer partition, with more precise signals, speculators could have been able to coordinate on attacking for a wider range of fundamentals, making a devaluation more likely.

When the government cannot commit to a disclosure policy, we find equilibria in which the government is made worse off by sending a public signal. Thus commitment is key for the government to benefit from disclosing information.

CHAPTER 2 : REPUTATION AND TRANSPARENCY

2.1. Introduction

I develop a model where short-term reputation concerns determine the public disclosure of information about the state of the economy, and then analyze its welfare effects in a coordination environment. When governments are privately informed about a payoff-relevant state, concerns for reputation might prevent them from truthfully disclosing the information. If the distribution of states is related to the government's type, the public information can be biased towards the state that is more likely under the agents' preferred type. The disclosure policy thus creates a bias in the agents' beliefs about the state, which affects their actions in equilibrium. When there is complementarity in the actions, it is possible that biased beliefs actually improve welfare in certain states.

There are *efficient* and *inefficient* governments, with privately known types, and both want to maximize their reputation for being efficient. The two types differ in their ability to generate the high and low productivity states. The high state is more likely when the government is efficient rather than inefficient. There is an underlying assumption that an unobservable and costly action can be taken to increase the probability of the high state, and only the efficient governments are willing to take that action. Governments learn the state and report it to entrepreneurs through a public signal. The reports are said to be *truthful* if they match the state, and they are *false* otherwise. The entrepreneurs rely on public information − the reports about the state and the realized productivity − to update their beliefs about the government.

Each period, entrepreneurs in the model can borrow in a competitive credit market to invest in a new venture, and there is complementarity in investment. Ventures face a common probability of failure, and entrepreneurs receive private signals about it. Conditional on success, the productivity of the ventures depends on the state of the economy. In equilibrium, given the public signal about the state, entrepreneurs follow a cutoff rule and invest if their private signals are high enough. The extent to which investment decisions respond to the public report depends on the government's reputation and on how truthful the public disclosure policy is. The more entrepreneurs believe that the state is high, the higher is their equilibrium cutoff given the public signal, and the more likely they are to invest.

In any equilibrium, the government's reputation evolves gradually over time, and entrepreneurs are never certain about the government's type − the distribution of productivity has the same support in both states. There is no equilibrium where the inefficient government follows a full disclosure policy. If the inefficient government were always truthful, the efficient government would respond by making false reports to distinguish itself from the inefficient type. This creates incentives for the inefficient government to deviate from full disclosure to be perceived as the efficient type. I focus on the equilibrium where the efficient government follows a full disclosure policy. In this equilibrium, the inefficient government is too optimistic: it is truthful in the high state, but in the low state it randomizes between true and false reports. The inefficient government's reports are thus biased toward the high state, which is more likely under the efficient type.

When the government reports a low state, entrepreneurs are certain that the state is indeed low, and their beliefs about the expected productivity are not biased. Following a report of a high state, entrepreneurs are not sure about the true state and their beliefs are biased: they overestimate productivity in the low state, and underestimate it in the high state. The higher is the trust in the government's announcement, the higher is the equilibrium level of investment when a report of a high state is sent. If the true state is high, welfare is increasing in the entrepreneurs' trust in the government, and welfare is maximized when entrepreneurs are sure of the state. However, in the low state there is a tradeoff when the inefficient government sends a false report: there are complementarity gains to all entrepreneurs from a higher level of investment, and potential losses to the marginal entrepreneurs due to the overestimation of productivity. As long as entrepreneurs do not place too much trust in the government's report, the bias is small enough and the net effect is positive for welfare.

When the trust in the public signal increases, false reports induce too much investment and reduce welfare in the low state.

Related literature.

This paper relates to the literature in which, due to reputation concerns, agents with privately known types may modify their actions to affect other agents' beliefs about their type (see Mailath and Samuelson (2001)). Here, I focus on the government's incentives to send optimistic signals to be perceived as an efficient type, even if that results in lower welfare. In contrast to what is commonly assumed in the literature of government reputation (see, for example, Barro and Gordon (1983) and Phelan (2006)), the government here cannot take actions that directly affect the agents' payoffs; it can only disclose information about payoff-relevant states, and actions cannot reveal the government's type. [1]

This paper is closely related to Herrera et al. (2015). They show that, for emerging economies, the rise in governments' popularity is a better predictor of financial crises than other better-known indicators, such as credit booms (see, for example, Mendoza and Terrones (2012) and Schularick and Taylor (2012)). The paper argues that governments in emerging economies are more concerned with their reputation and choose to enjoy the short-term popularity benefits of weak credit booms rather than implementing costly policies that would reduce the probability that such booms end in crises. They develop a model where booms can be good (sustainable) or bad (unsustainable), and the policy that maximizes welfare is the regulation of bad booms, and no regulation of good booms. There is a good government, which always acts optimally, and a bad government that is strategic and wants to maximize its reputation for being good. Since the good boom is more likely under the good government, the bad governments will not always regulate bad booms, as regulation reveals that the boom is not sustainable and it negatively impacts the government's reputation. In my model, I assume that the government cannot directly affect the

[1] Here, the government is not trying to convince agents that it will not behave *opportunistically* and take an action that negatively affects their payoffs (such as increasing capital taxes). Instead, the government is trying to show the agents that it can generate the high productivity states more often.

outcome in the economy; the welfare effects of the public disclosure policy depend on how agents respond to it. As in Herrera et al. (2015), a large increase in reputation can be a bad sign for welfare in my model. When the inefficient government sends false reports with high probability, agents do not trust reports of a high productivity state. Thus, if the government sends a false report, it is not believed and there is only a small rise in reputation, followed by a small increase in credit and investment, which is welfare improving because of the complementarity in investment. However, when the inefficient government is not likely to make false reports, agents trust the public signal. In this case, following a false report in the low productivity state, there is large increase in reputation and a high level of investment, which decreases welfare. If entrepreneurs are required to borrow in order to invest, this results in a high probability of default, which can be interpreted as a credit crisis.

The paper also relates to the literature on pandering. For instance, Maskin and Tirole (2004) analyze a model where politicians might have the same preferences as the electorate or not, and their type is privately known. They show that when a politician has strong motives to remain in office, she always takes the popular action (the *ex ante* optimal action for the voters), even if she knows that the action is not optimal, and regardless of her type. The politician thus panders to public opinion because she wants to build a reputation for being the type that has the same preferences as the voters. In a different setting, a similar result is presented in Brandenburger and Polak (1996). They show that when a manager is concerned with the market's perception about his actions, he will distort his investment decision toward an investment that the market believes is *ex ante* more likely to succeed. In my model, instead of a privately informed government making decisions, I have agents that choose their actions based a public signal, and the government's type affects the distribution of payoff relevant states. The disclosure policy follows the same logic of pandering: when the efficient government is truthful, the inefficient government sends signals that are biased toward the state that agents believe is more likely when the government is efficient.

Finally, the paper is related to the literature on global games of regime change. The model can be interpreted as having two regimes: the default is a low productivity regime; and if the level of investment is high enough, there is a switch to a high productivity regime. By investing, entrepreneurs are attacking the low productivity regime, and the probability of failure is assumed to affect the success of the attack. In each period, the game between entrepreneurs is similar to the one in Morris and Shin (1998), who study a model of self-fulfilling currency attacks when the fundamentals that determine payoffs are not common knowledge among entrepreneurs. The equivalent to their state of the fundamentals in my model is the venture's probability of failure. As in Morris and Shin (1998), the game between entrepreneurs has a unique equilibrium when the noise in the entrepreneurs' private information is small enough, and the equilibrium investment strategies also follows a cutoff rule based on their private signals. Deviation from common knowledge is key for the uniqueness of equilibrium. My model departs from Morris and Shin (1998) by introducing another state variable that affects the payoffs in case of a regime change, and a government that sends public signals about that variable (the state of the economy in the current paper, which affects the productivity of the ventures). The introduction of public policy in such coordination environments, and its signaling effects, have been extensively studied in the literature (see, for example, Angeletos et al. (2006), Angeletos and Pavan (2007, 2009) and Angeletos and Pavan (2013)). Breaking the uniqueness result in Morris and Shin (1998), Angeletos et al. (2006) point out that policy interventions that convey some information about the fundamentals may lead to multiple equilibria. In contrast to Angeletos et al. (2006), the public policy in my model does not lead to multiplicity. This is the case because there is no public information about the state that affects the success of an attack, only about the state that determines payoffs conditional on the regime change. The public signal only affects the entrepreneurs' cutoff rule: the cutoff is increasing in the probability that entrepreneurs assign to the high productivity state.

Structure of the paper. The remainder of this paper is divided as follows. Section 2.2 presents the model and the equilibrium disclosure policy for the government is characterized in Section 2.3. Section 2.4 analyzes the entrepreneurs' equilibrium investment strategies, and the welfare effects of the public policy. Section 2.5 concludes the paper and discusses extensions to the model. Appendix A.4 analyzes the model when entrepreneurs are require to borrow in a competitive credit market to star a new venture. Furthermore, it presents conditions under which the two models have the same equilibrium investment strategies for the entrepreneurs. The proofs that are omitted in the main text are presented in Appendix A.3.

2.2. The Model

2.2.1. Actions and payoffs

Time is discrete and indexed by $t \in \{1, 2, ...\}$. There is a continuum of entrepreneurs of measure one, who are indexed by i and uniformly distributed on $[0, 1]$. They are infinitely-lived, risk-neutral profit maximizers. Each period, entrepreneurs have an endowment of one unit of labor, which can be used to start up a new, risky, venture, or to work for a fixed wage w.[2] For simplicity, there is no capital in the model, only labor. Appendix A.4 analyzes the model when new ventures also require one unit of capital, which is borrowed in a perfectly competitive credit market. In the model with capital, there is an equilibrium where the investment decisions are the same as in the baseline model without capital. [3]

The ventures have a common probability of failure θ_t, which is drawn every period from a uniform distribution on $\Theta = [\theta_{\min}, \theta_{\max}]$. The total number of ventures in period t is denoted by n_t. In case of success, at the end of period t the venture pays

$$v, \quad \text{if } n_t < N(\theta_t),$$

[2]The wage w can be seen as the payoff from choosing a safe rather than a risky venture.

[3]For that result, the opportunity cost of a venture must be the same in both models. Without capital, the opportunity cost is w, the cost of labor. With capital, the opportunity cost is $1 + r + \tilde{w}$, the cost of labor plus capital, where r and $\tilde{w}$ are the risk-free rate and the wage in the model with capital. Therefore, we need that $w = 1 + r + \tilde{w}$.

and

$$v + \delta_t, \quad \text{if } n_t \geq N(\theta_t),$$

where $N(\cdot) < 1$ is weakly increasing in θ, with a continuous derivative $N'(\cdot)$. The productivity of the ventures is thus increased by $\delta_t > 0$ if the aggregate investment is high enough. Failed ventures are assumed to pay nothing.

Each period, the productivity parameter δ depends on a state variable $s \in S = \{H, L\}$. Given s_t, δ_t is follows a distribution with probability density function f_{s_t} and mean δ_{s_t}. It is assumed that

$$\text{supp} f_H = \text{supp} f_L = \Delta = [\delta_{\min}, \delta_{\max}],$$

in which case the realization of δ never reveals the state. State H is associated with higher productivity, as described in the assumption below.

Assumption 2 *The likelihood ratio $\lambda(\delta) \equiv f_H(\delta)/f_L(\delta)$ is continuously differentiable, increasing in δ, and it is strictly increasing for $\delta \in (\delta_1, \delta_2) \subseteq [\delta_{min}, \delta_{max}]$, where $\delta_1 < \delta_2$.*

Assumption 1 implies that $\delta_H > \delta_L$.

There is also a government in this economy, which can be *efficient* (type E) or *inefficient* (type I), and the types are private information. It is assumed that the type is permanent and the same government remains in power forever. The types only differ in their ability to generate the high productivity state H. Each period, high productivity states are more likely when the government is efficient:

$$\pi_E \equiv Pr(s_t = H|E) > \pi_B \equiv Pr(s_t = H|I), \quad \text{for all } t.$$

The government knows the state and can report it through a public signal $y_t \in Y = \{h, l\}$.[4]

[4]There are two interpretations for the disclosure policy. One is that the government observes s_t and sends a (possibly random) public signal $y(s_t)$. Another interpretation is that the government follows an information acquisition procedure and, if the state is s_t, the outcome is a (possibly random) public signal $y(s_t)$. The latter is in line with the Bayesian persuasion literature (see, for example, Kamenica and Gentzkow

We say that the report is *truthful* if either $y_t = h$ when $s_t = H$, or $y_t = l$ when $s_t = L$, and it is *false* otherwise. The government's reputation at the beginning of period t is denoted by μ_t, which is the probability that entrepreneurs assign to the efficient type E. The government's payoff at period t is given by μ_{t+1}, the updated reputation at the end of the period, after entrepreneurs observe y_t and δ_t. Both types, E and I are strategic, and their goal in each period is to maximize the expected value of μ_t. The governments are assumed to be myopic and only care about their reputation at the end of the period.[5]

2.2.2. Timing and information

At period $t = 1$, nature draws the government's type from $\{E, I\}$ Entrepreneurs enter period $t = 1$ with a common prior μ_1 about the government's type. At the beginning of period t, nature draws the probability of failure $\theta_t \in \Theta$ and the state $s_t \in \{H, L\}$. The government observes s and sends a public signal $y_t \in \{h, l\}$ about the state. Entrepreneurs then form beliefs about the state and the expected value of δ_t. The expected value of δ_t is $\bar{\delta}_t = \mathbb{P}(s = H|\mu_t, y_t)\delta_H + \mathbb{P}(s = L|\mu_t, y_t)\delta_L \in [\delta_L, \delta_H]$. Entrepreneur i also receives a private signal $x_{t,i}$ about θ_t. After observing the private and public signals, entrepreneurs simultaneously decide whether to invest or to work. Given s_t, nature draws the productivity parameter δ from a distribution with probability density function f_{s_t}. At the end of the period, the outcomes of all ventures are publicly observed, payoffs are received, and the government's reputation is updated to μ_{t+1}. The structure of the game is assumed to be common knowledge.

Given θ_t, entrepreneur i receives a private signal $x_{t,i} \in X = [\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon]$, where

$$x_{t,i} = \theta_t + \varepsilon_{t,i}.$$

(2011)). In this case, the government is not more informed than the entrepreneurs when the public signal is sent.

[5] The government in this model can be seen as a party (efficient or inefficient) that is perpetually in power. Each period there is a different member of the party who runs the government (the current president). She only cares maximizing the reputation of the party while she is charge, and future reputation is not a concern. Extensions where the government can be replaced and care about future reputation are discussed Section 2.5.

The idiosyncratic noise $\varepsilon_{t,i}$ is drawn from a distribution with probability density function $g(\cdot)$, and cumulative distribution function $G(\cdot)$. Each $\varepsilon_{t,i}$ is independently and identically distributed across entrepreneurs and independent of θ_t. I assume that $\text{supp}(g) = [-\varepsilon, \varepsilon]$, with $\varepsilon > 0$, and $2\varepsilon < \min\{\underline{\theta} - \theta_{\min}, \theta_{\max} - \bar{\theta}_{\delta_H}\}$. Function $g(\cdot)$ is differentiable on $(-\varepsilon, \varepsilon)$, and its derivative, $g'(\cdot)$, is assumed to be bounded and such that[6]

$$\text{if } g'(\tilde{\varepsilon}) < 0, \text{ then } g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} \in (\tilde{\varepsilon}, \varepsilon). \tag{2.1}$$

The posterior distribution of θ given private signal x has probability density function $\phi(\theta|x)$, where

$$\phi(\theta|x) = \frac{g(x-\theta)}{G\left(x - \theta_{\min}\right) - G\left(x - \theta_{\max}\right)}. \tag{2.2}$$

The derivation of $\phi(\theta|x)$ is presented in Appendix A.2.[7]

At the end of the period, entrepreneurs might observe the realization of δ and use it to update their beliefs about the government. There are two alternative frameworks.

Assumption 3-A *At the end of the period, the realization of δ is always publicly observed.*

Assumption 3-B *The realization of δ is publicly observed when $n \geq N(\theta)$, in which case successful ventures pay $v + \delta$. If $n < N(\theta)$, entrepreneurs do not observe δ.*

In what follows, the results are true under both Assumption 3-A or Assumption 3-B, unless the required assumption is clearly specified.

2.2.3. Equilibrium

I restrict attention to *Markov* strategies: for any t, t', if $\mu_t = \mu_{t'}$, the government and the entrepreneurs follow the same strategies in periods t and t'. In other words, conditional on

[6] The assumptions on the structure of private signals are based on a previous work Galvao and Shalders (2017). They guarantee that, conditional on the public signal, the game between entrepreneurs in each period has a unique equilibrium.

[7] There is a pair of values of x that fully reveals θ. If $x = \theta_{\max} + \varepsilon$, we have $\mathbb{P}(\theta = \theta_{\max}|x = \theta_{\max} + \varepsilon) = 1$; likewise, when $x = \theta_{\min} - \varepsilon$, then $\mathbb{P}(\theta = \theta_{\min}|x = \theta_{\min} - \varepsilon) = 1$. For all other values of x, the conditional density of θ is given by (2.2).

the current beliefs about the government, the strategies are independent of the history of actions, states, and outcomes that lead to those beliefs.

Remark: In this paper, the link between periods is the evolution of entrepreneurs' beliefs about the government. The per-period payoffs are independent of past and future actions, states and outcomes, the government maximizes its expected reputation at the end of each period, and I limit attention to equilibria in Markov strategies. I chose this highly stylized dynamic game rather than a static one to capture the evolution of the government's reputation, and how it affects entrepreneurs' strategies. In Section 2.5 I discuss possible extensions that would make the dynamic game more realistic.

The efficient government's strategy for period t is denoted by $p_E : [0,1] \times S \to [0,1]$, where $p_E(\mu_t, s)$ is the probability that the efficient government sends a signal $y = h$, given that the prior reputation is μ_t and the current state is s. [8] Similarly, the inefficient government's strategy for period t is denoted by $p_I : [0,1] \times S \to [0,1]$. Entrepreneurs beliefs about the productivity parameter δ are given by $\bar{\delta} : [0,1] \times Y \to [\delta_L, \delta_H]$, where $\bar{\delta}(\mu_t, y)$ is the expected value of δ given that a government with reputation μ_t has sent a public signal y. Entrepreneur i's strategy for period t is given by $a_i : [\delta_L, \delta_H] \times X \to \{0, 1\}$, where $a_i(\bar{\delta}, x_i) = 1$ represents investing and $a_i(\bar{\delta}, x_i) = 0$ represents working, given a private signal x_i and $\bar{\delta}$.[9] The government's reputation at the end of period t is given by $\mu_{t+1} : [0,1] \times Y \times \Theta \times \Delta$, where $\mu_{t+1}(\mu_t, y, \theta, \delta)$ is the probability that entrepreneurs assign to the efficient type if a government of reputation μ_t sends a signal y, and the observed productivity is δ.

The equilibrium concept here is perfect Bayesian equilibrium (PBE). Given a common prior μ_1, a PBE consists of entrepreneurs' beliefs μ_t, strategies for types E, p_E, for type I, p_I, and for the continuum of entrepreneurs, $\{a_i\}_{i \in [0,1]}$, such that beliefs are updated using Bayes rule whenever possible[10] and, given the beliefs, no player has an incentive to deviate.

[8] Given the restriction to Markov strategies, I drop the subscript t, except for the reputation μ_t.

[9] It is assumed that entrepreneurs invest whenever indifferent, thus $a_i \in \{0, 1\}$.

[10] In this setting, government's deviations from equilibrium are only observable if, for a prior reputation μ_t, both types send either $y_t = h$ or $y_t = l$ with probability 1, regardless of the true state s_t. Apart from the case of observable deviations, entrepreneurs use Bayes rule to update their beliefs.

2.3. Optimal Disclosure Policy

This section characterizes the equilibrium disclosure policies at period t. At the beginning of period t, the prior reputation is given by μ_t, the probability that entrepreneurs assign to the efficient type at the end of period $t-1$. The efficient government's strategy for period t is given by $p_E(\mu_t, s)$, which denotes the probability that type E sends a signal $y = h$ given μ_t and state s. Similarly, the inefficient type I's strategy is given by $p_I(\mu_t, s)$. Both types follow disclosure policies that maximize their expected reputation at the end of the period, μ_{t+1}.

An equilibrium profile for time t consists of strategies p_E and p_I for types E and I, and beliefs and strategies for the entrepreneurs, such that beliefs are obtained using Bayes rule whenever possible[11] and, given the beliefs, no player has an incentive to deviate. This section characterizes the entrepreneurs' beliefs and the equilibrium strategies for the government. The equilibrium strategies for the entrepreneurs are characterized in Section 2.4.

There exist the trivial equilibria in which both types send either $y = h$ or $y = l$ regardless of the state. These equilibria are supported by the belief that the government is inefficient whenever a deviation is observed. There is no equilibrium in which the inefficient type I follows a full disclosure policy, i.e., where the reports are always truthful: $y = h$ in state H, and $y = l$ in state L. This result is formalized in Lemma 19, in Appendix A.2. Intuitively, if the inefficient government were always truthful, the efficient government would respond by making false reports to distinguish itself from the inefficient type. This creates incentives for the inefficient government to deviate from full disclosure to be perceived as the efficient type.

There exist equilibria where the efficient government follows a full disclosure policy. In what follows, I restrict attention to such equilibria. First, the efficient government is assumed to

[11] See footnote 10

follow

$$p_E(\mu_t, H) = 1 - p_E(\mu_t, L) = 1, \quad \text{for all } \mu_t.$$

Then the best response of the inefficient government is characterized. Finally, I check whether this is an equilibrium strategy profile for period t.

2.3.1. Reputation

Given μ_t and the governments' strategies, entrepreneurs update beliefs using Bayes rule. First, entrepreneurs form intermediate beliefs following the public signal y and make investment decisions. Then, conditional on observing a realization of δ, entrepreneurs update the reputation to μ_{t+1}. If the government sends a public signal $y = h$, the entrepreneur's intermediate update about the government's reputation is

$$\mu^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \left[\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)\right](1 - \mu_t)}. \tag{2.3}$$

If $y = l$, the intermediate update is

$$\mu^l(\mu_t) = \frac{(1 - \pi_E)\mu_t}{(1 - \pi_E)\mu_t + \left[\pi_I(1 - p_I(\mu_t, H)) + (1 - \pi_I)(1 - p_I(\mu_t, L))\right](1 - \mu_t)}. \tag{2.4}$$

Given the public signal y, entrepreneurs form beliefs about the expected value of δ. If $y = h$, the expectation of δ is

$$\begin{aligned}\bar{\delta}(\mu_t, h) = & \left[\mu^h(\mu_t) + (1 - \mu^h(\mu_t)) \frac{\pi_I p_I(\mu_t, H)}{\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)}\right] \delta_H \\ & + \left[(1 - \mu^h(\mu_t)) \frac{(1 - \pi_I) p_I(\mu_t, L)}{\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)}\right] \delta_L,\end{aligned} \tag{2.5}$$

and if $y = l$,

$$\bar{\delta}(\mu_t, l) = \left[(1-\mu^l(\mu_t))\frac{\pi_I(1-p_I(\mu_t,H))}{\pi_I(1-p_I(\mu_t,H)) + (1-\pi_I)(1-p_I(\mu_t,L))}\right]\delta_H + \left[\mu^l(\mu_t) + (1-\mu^l(\mu_t))\frac{(1-\pi_I)(1-p_I(\mu_t,L))}{\pi_I(1-p_I(\mu_t,H)) + (1-\pi_I)(1-p_I(\mu_t,L))}\right]\delta_L. \quad (2.6)$$

After investment decisions are made and the outcomes of all ventures are observed, entrepreneurs might observe δ. If $y = h$ and δ is observed, the government's updated reputation is

$$\mu_\delta^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \left[\pi_I p_I(\mu_t, H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_I)p_I(\mu_t, L)\right](1-\mu)}. \quad (2.7)$$

From Assumption 2, the likelihood ratio $f_H(\delta)/f_L(\delta)$ is increasing in δ, thus μ_δ^h is also increasing in δ. The higher is δ, the more likely it is that the true state is H and that the report $y = h$ is truthful. Since type E is always truthful, the reputation increases in δ.

If $y = l$ and δ is observed, the updated reputation is

$$\mu_\delta^l(\mu_t) = \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + \left[\frac{f_H(\delta)}{f_L(\delta)}\pi_I(1-p_I(\mu_t,H)) + (1-\pi_I)(1-p_I(\mu_t,L))\right](1-\mu_t)}. \quad (2.8)$$

From Assumption 2, μ_δ^l is decreasing in δ. As δ increases, it less likely that the true state is L and that the report $y = l$ is truthful, therefore the reputation decreases.

Under Assumption 3-A, the realization of δ is always observed at the end of the period. In this case, the government's reputation at the end of the period is given by

$$\mu_{t+1}(\mu_t, y, \theta, \delta) = \mu_\delta^y(\mu_t), \quad \text{for all } \mu_t, y, \theta, \delta. \quad (2.9)$$

Given prior reputation μ_t and state s, the expected reputation by sending signal y is

$$\bar{\mu}_t(\mu_t, s, y) = \mathbb{E}_\delta[\mu_{t+1}(\mu_t, y, \theta, \delta)|s]. \quad (2.10)$$

Under Assumption 3-B, the realization of δ is only observed when the number of ventures is greater than $N(\theta)$, in which case successful ventures pay $v + \delta$. After observing y, entrepreneurs compute the expected value of δ, $\bar{\delta}(\mu_t, y)$, observe their private signals x_i and make their investment decisions, which are characterized in Section 2.4. From Proposition 4 in Section 2.4, the higher is the entrepreneurs expectation of δ, the higher is the equilibrium number of ventures, n, and the higher is the probability that δ is observed. It follows from Proposition 4 in Section 2.4 that, under Assumption 3-B, the government's reputation at the end of the period is given by

$$\mu_{t+1}(\mu_t, y, \theta, \delta) = \begin{cases} \mu^y_\delta(\mu_t), & \text{if } \theta \leq \theta^*(\bar{\delta}(\mu_t, y)) \\ \mu^y(\mu_t), & \text{if } \theta > \theta^*(\bar{\delta}(\mu_t, y)) \end{cases} . \tag{2.11}$$

The probability of δ being observed is $\mathbb{P}(\theta \leq \theta^*(\bar{\delta}(\mu_t, y))) \equiv P^*(\mu_t, y)$. Given a prior μ_t and a state s, the government's expected reputation from sending a signal y is

$$\bar{\mu}_t(\mu_t, s, y) = P^*(\mu_t, y)\mathbb{E}_\delta[\mu^y_\delta(\mu_t)|s] + [1 - P^*(\mu_t, y)]\mu^y(\mu_t). \tag{2.12}$$

The government's objective is to maximize $\bar{\mu}_t$. The expected payoff gain from being truthful in state H and sending a signal h rather than a signal l is given by

$$G_H = \bar{\mu}_t(\mu_t, H, h) - \bar{\mu}_t(\mu_t, H, l). \tag{2.13}$$

The gain from being truthful in state L is given by

$$G_L = \bar{\mu}_t(\mu_t, L, l) - \bar{\mu}_t(\mu_t, L, h). \tag{2.14}$$

2.3.2. Equilibrium policy

In any equilibrium where the efficient type follows a full disclosure policy, the inefficient type will truthfully disclose the high productivity state H, as stated in the lemma below.

Lemma 9 *Let $\mu_t = \mu \in (0,1)$. If the efficient government follows full disclosure, then the inefficient government is truthful when $s = H$:*

$$p_I(\mu, H) = 1.$$

The proof is in Appendix A.2. Intuitively, there are two reasons for the inefficient government to be truthful in state H when the efficient government is always truthful. When entrepreneurs observe a signal h, they believe that it is more likely that the government is efficient, since state H is more likely when the government is efficient. The second reason is that, if the government sends $y = l$ and entrepreneurs observe a realization of δ that is more likely under state H, they will assign a high probability to a false report, which only happens if the government is inefficient. Thus, by sending a signal h, the inefficient government increases both its reputation prior to the realization of δ and the expected reputation conditional on δ being observed.

Since the inefficient government is truthful in the high productivity state, there can only be false reports in the low productivity state. In what follows, denote by p_μ the probability that the inefficient government sends a signal h in state L ($p_\mu \equiv p_I(\mu, L)$). We can write the gain from making truthful reports in state L, given by (2.14), as a function $G_L(\mu, p_\mu)$.

Lemma 10 *$G_L(\mu, p_\mu)$ has the following properties:*

(i) $G_L(0, p) = G_L(1, p) = 0, \quad$ *for all* $p \in [0, 1]$.

(ii) $G_L(\mu, 0) < 0$, *for all* $\mu \in (0, 1)$.

(iii) $G_L(\mu, 1) > 0$, *for all* $\mu \in (0, 1)$.

(iv) Under Assumption 3-A, $G_L(\mu, 0)$ *is strictly convex in* μ.

From part (i) of Lemma 10, when entrepreneurs are sure about the government's type, the inefficient government has no incentives to make false reports in the low productivity state.

However, from part (ii), incentives arise when there is uncertainty about the government's type. Part (iii), shows that the incentives to lie disappear when the probability of false reports, p_μ, becomes too high. Finally, parts (i), (ii), and (iv) imply that, under Assumption 3-A, the gain from always being truthful in state L, $G_L(\mu, 0)$, is U-shaped in μ: starting from $G_L(0,0) = 0$, $G_L(\mu, 0)$ first decreases in μ, then it increases to reach $G_L(1,0) = 0$. The incentives for the inefficient government to make false reports are thus highest for intermediate values of the prior reputation.

From Lemma 10 we get the following result.

Lemma 11 *Let $\mu_t = \mu \in (0,1)$. If the efficient type follows full disclosure, then the inefficient government sends $y = l$ with positive probability in state L:*

$$p_\mu \in (0,1),$$

where p_μ is such that $G_L(\mu, p_\mu) = 0$. If Assumption 3-A holds, there exists a unique $p^*_\mu \in (0,1)$ that solves $G_L(\mu, p_\mu) = 0$.

Given the inefficient government's response to the efficient government's full disclosure policy, it is left to show that the efficient government has no incentives to deviate, and that the strategy profile is indeed an equilibrium for period t. This result is described in the following proposition.

Proposition 3 *Let $\mu_t = \mu \in (0,1)$. There exists an equilibrium where, in period t, the efficient government follows a full disclosure policy and the inefficient government sets*

$$p_I(\mu_t, H) = 1,$$

and

$$p_I(\mu, L) = p_\mu \in (0,1),$$

where p_μ solves $G_L(\mu, p_\mu) = 0$.

If Assumption 3-A holds, there exists a unique p_μ^* that solves $G_L(\mu, p_\mu) = 0$.

2.4. Investment and Welfare

This section analyzes the entrepreneurs' equilibrium strategies for period t, and the equilibrium levels of investment and welfare. Given a public signal y and prior reputation μ_t, entrepreneurs form expectations about δ, as described in (2.5) and (2.6), to make investment decisions. Here, I fix the expected value of δ at $\bar{\delta}(\mu_t, y) = \bar{\delta}$.

2.4.1. Investment

As mention in Section 2.2.3, given the restriction to Markov strategies, the entrepreneur i's strategy only depends on the current private signal x_i and on $\bar{\delta}$. Hence, conditional on $\bar{\delta}$, the game between the entrepreneurs in each period is similar to the one in Morris and Shin (1998). In their paper, entrepreneurs decide whether to attack a currency or not based on their private signals about the fundamentals of the economy. In the current paper, given $\bar{\delta}$, entrepreneurs decide whether to invest or to work given their private signals about the venture's probability of failure. For a given probability of failure θ, and a given $\bar{\delta}$, an entrepreneur's expected payoff from investing is

$$(1-\theta)v, \quad \text{if } n < N(\theta),$$

and

$$(1-\theta)(v+\bar{\delta}), \quad \text{if } n \geq N(\theta).$$

Denote by $\underline{\theta}$ the value of θ that solves $(1-\theta)v = w$. If $\theta < \underline{\theta}$, it is optimal to invest even if no other entrepreneur is investing. Denote by $\bar{\theta}(\bar{\delta})$ the value of θ that solves $(1-\theta)(v+\bar{\delta}) = w$. If $\theta > \bar{\theta}(\bar{\delta})$, it is not optimal to invest even if all entrepreneurs are investing. To simplify the notation, let $\bar{\theta}_H \equiv \bar{\theta}(\delta_H)$, and $\bar{\theta}_L \equiv \bar{\theta}(\delta_L)$.

When there is common knowledge about the probability of failure, Θ can be divided in three intervals[12], as is standard in the literature of self-fulfilling equilibria:[13]

- if $\theta \in [\theta_{\min}, \underline{\theta})$: it is always profitable to invest;
- if $\theta \in (\underline{\theta}, \bar{\theta}(\bar{\delta}))$: coordinated investment is profitable and, if entrepreneurs coordinate on not investing, investment is not profitable;
- if $\theta \in (\bar{\theta}(\bar{\delta}), \theta_{\max}]$: it is never profitable to invest.

As the expected value of δ, $\bar{\delta}$, increases, the threshold $\bar{\theta}(\bar{\delta})$ also increases. This means that there are more values of θ for which coordinated investment is profitable (the middle interval grows to the right), and there are fewer values of θ that prevent investment from being profitable (the upper interval shrinks).

Now we turn to the equilibrium with private information about θ. Conditional on $\bar{\delta}$, an equilibrium for the game between the entrepreneurs in period t consists of strategies such that no entrepreneur has an incentive to deviate. For a given profile of strategies for the entrepreneurs, the measure of entrepreneurs who invest given $\bar{\delta}$ and a private signal x is denoted by $\eta(\bar{\delta}, x)$. Given a probability of failure θ, the number of ventures is then

$$n(\bar{\delta}, \theta, \eta) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \eta(\bar{\delta}, x) g\left(x - \theta\right) dx. \tag{2.15}$$

Conditional on success, the expected productivity of a venture is increased by $\bar{\delta}$ when

$$n(\bar{\delta}, \theta, \eta) \geq N(\theta). \tag{2.16}$$

[12]It is assumed that

- $v > w$;
- $\underline{\theta} = 1 - w/v > \theta_{\min}$;
- $\bar{\theta}_H = 1 - w/[v + \delta_H] < \theta_{\max}$.

[13]See, for example, Obstfeld (1996) and Morris and Shin (1998) in the case of self-fulfilling currency attacks.

Thus, the event where a venture's expected payoff is $v + \bar{\delta}$ is given by

$$A(\bar{\delta}, \eta) = \{\theta : n(\bar{\delta}, \theta, \eta) \geq N(\theta)\}. \tag{2.17}$$

After observing x_i, entrepreneur i's expected payoff from investing is:

$$u(\bar{\delta}, x_i, \eta) = v \int_{x_i-\varepsilon}^{x_i+\varepsilon} (1-\theta)\phi(\theta|x_i)d\theta + \bar{\delta} \int_{[x_i-\varepsilon, x_i+\varepsilon]\cap A(\bar{\delta},\eta)} (1-\theta)\phi(\theta|x_i)d\theta, \tag{2.18}$$

where ϕ is given by (2.2). Entrepreneur i invests in equilibrium if:

$$u(\bar{\delta}, x_i, \eta) \geq w. \tag{2.19}$$

The following proposition characterizes the unique equilibrium of the game played by the entrepreneurs at time t, conditional on $\bar{\delta}$.

Proposition 4 *Given $\bar{\delta}$, the equilibrium of the game between entrepreneurs in period t is unique. The equilibrium strategy for the entrepreneurs is to invest if and only if their private signal is*

$$x \leq x^*(\bar{\delta}).$$

The equilibrium number of ventures n is thus decreasing in θ. $n \geq N(\theta)$ if and only if the probability of failure is

$$\theta \leq \theta^*(\bar{\delta}).$$

Both $x^*(\bar{\delta})$ and $\theta^*(\bar{\delta})$ are increasing in $\bar{\delta}$.

The proof of Proposition 4 is in Appendix A.3. Entrepreneurs follow a cutoff rule and invest if their private signal is below $x^*(\bar{\delta})$. Since $x^*(\bar{\delta})$ is increasing, for every θ the number of ventures is increasing in the entrepreneurs' expectation of δ. The cutoff rule leads to a threshold probability of failure $\theta^*(\bar{\delta})$, below which the total number of ventures is greater than $N(\theta)$, and the successful ventures pay $v + \delta$ instead of v. Since the threshold $\theta^*(\bar{\delta})$ is also increasing, the higher is the entrepreneurs' expectation of δ, the higher is the probability that ventures pay $v+\delta$ instead of v.

The entrepreneurs' equilibrium strategy is thus

$$a_i(\bar{\delta}, x_i) = a^*(\bar{\delta}, x_i) = \begin{cases} 1, & \text{if } x_i \leq x^*(\bar{\delta}) \\ 0, & \text{if } x_i > x^*(\bar{\delta}) \end{cases} . \tag{2.20}$$

where $x^*(\bar{\delta})$ solves

$$u(\bar{\delta}, x^*(\bar{\delta}), a^*) = w, \tag{2.21}$$

Equation (2.21) is the indifference condition for the entrepreneur who receives the cutoff signal $x^*(\bar{\delta})$. In equilibrium, the total number of ventures is given by

$$n(\bar{\delta}, \theta, a^*) = \mathbb{P}(x \leq x^*(\bar{\delta})|\theta) = G(x^*(\bar{\delta}) - \theta).$$

2.4.2. Welfare

In state s, the mean value of δ is δ_s. If $\bar{\delta} \neq \delta_s$, the entrepreneurs' expectation of the productivity parameter is biased. The entrepreneurs' expected welfare in state s is given by[14]

$$\begin{aligned} W_s(\bar{\delta}) =& (v+\delta_s)\int_{\theta_{\min}}^{\theta^*(\bar{\delta})} (1-\theta)G(x^*(\bar{\delta})-\theta)d\theta + v\int_{\theta^*(\bar{\delta})}^{x^*(\bar{\delta})+\varepsilon} (1-\theta)G(x^*-\theta)d\theta \\ &+ w\left[\int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1-G(x^*(\bar{\delta})-\theta))d\theta + \int_{x^*(\bar{\delta})+\varepsilon}^{\theta_{\max}} d\theta\right]. \end{aligned} \tag{2.22}$$

From Proposition 3, both types are truthful when the state is H, but the inefficient government sends false reports with positive probability in state L. Following a signal $y = l$, entrepreneurs are sure that the true state is L, and there is no distortion in the entrepreneurs'

[14] θ is uniformly distributed on $[\theta_{\min}, \theta_{\max}]$, therefore the density is constant at $[1/(\theta_{\max} - \theta_{\min})]$. For simplicity, I multiplied the welfare function by $[\theta_{\max} - \theta_{\min}]$.

expectation about δ: $\bar{\delta}(\mu, l) = \delta_L$ for all $\mu \in (0, 1)$. However, when the government sends a signal $y = h$, there is a distortion: $\bar{\delta}(\mu, l) \in (\delta_L, \delta_H)$ for all $\mu \in (0, 1)$. Entrepreneurs overestimate δ when the true state is L, and underestimate δ when the state is H. The higher is the entrepreneurs' trust in the public signal $-$ their belief that the report is truthful, and the state is H $-$ the higher is $\bar{\delta}$. Hence, in state L the distortion increases with the entrepreneurs' trust a signal h ($\bar{\delta}$ gets further away from δ_L), while in state H the distortion decreases with the entrepreneurs' trust ($\bar{\delta}$ gets closer to δ_L).

If the true state is H, welfare is increasing in the entrepreneurs' expectation of δ, and it is maximized at $\bar{\delta} = \delta_H$ (i.e., when the expectation is unbiased). This result is stated in the following lemma.

Lemma 12 *$W_H(\bar{\delta})$ in increasing in $\bar{\delta}$, for all $\bar{\delta} \leq \delta_H$.*

The more entrepreneurs believe that the government is being truthful when sending $y = h$, the higher is their expectation of δ and the more they are willing to invest. Lemma 12 thus implies that, in the high productivity state, welfare is increasing in the entrepreneurs' trust in the public signal.

In the low productivity state L, welfare increases if the entrepreneurs' expectation of δ is slightly biased. Starting at $\bar{\delta} = \delta_L$, a marginal increase in the $\bar{\delta}$ increases W_L. This result is formalized in the following lemma.

Lemma 13 *$\frac{\partial W_L(\bar{\delta})}{\partial \bar{\delta}} > 0$, at $\bar{\delta} = \delta_L$.*

Lemma 13 shows that entrepreneurs might benefit from having biased expectation of δ in state L. Biased expectations induce entrepreneurs to be more aggressive in their investment strategies and receive the complementarity gain δ more often. Complementarity in investment is thus key to this result. However, as the bias increases, welfare might start to decrease. This is the case when $x^*(\delta_H) > \bar{\theta}_L + \varepsilon$, which is true if $(\delta_H - \delta_L)$ is large enough.[15]

[15] For example, if $\delta_H > \frac{2\varepsilon(v+\delta_L)^2}{w-2\varepsilon(v+\delta_L)}$.

This result is presented in the following lemma.

Lemma 14 *Suppose that $x^*(\delta_H) > \bar{\theta}_L + \varepsilon$. Then, there exists $\tilde{\delta} \in (\delta_L, \delta_H)$ such that $\frac{\partial W_L(\bar{\delta})}{\partial \bar{\delta}} < 0$, for $\bar{\delta} \geq \tilde{\delta}$.*

The intuition for Lemmas 13 and 14 is the following. When $\bar{\delta}$ increases, entrepreneurs expected payoff from investing also increases. This raises the equilibrium cutoff signal for investing, $x^*(\bar{\delta})$, which in turn raises the threshold $\theta^*(\bar{\delta})$, below which entrepreneurs receive the productivity gain δ. When the true state is L, there is a tradeoff from raising the cutoff: the marginal investors are worse off due to their biased expectation of δ; while all entrepreneurs gain from the a higher level of investment. In equilibrium, there is more investment when the probability of failure is low ($\theta < \bar{\theta}_L$), and it is optimal to invest, but there there is also more investment when the probability of failure is high ($\theta > \bar{\theta}_L$), and it is optimal to work. If the entrepreneurs' expectation is biased, but close enough to $\bar{\theta}_L$, the positive effect dominates, and raising the cutoff increases welfare W_L. However, when the entrepreneurs' expectation of δ is too biased, such that $x^*(\bar{\delta}) > \bar{\theta}_L + \varepsilon$, the tradeoff disappears and only the negative effect on W_L remains: raising the cutoff only increases investment when $\theta > \bar{\theta}_L$, and it is optimal to work.

Thus, when the true state is L, if entrepreneurs assign a small probability to state H, there is a small increase in investment, which is welfare improving. As entrepreneurs become more convinced that the state H when it is in fact L, welfare starts to decrease because there is too much investment when the probability of failure is high, and working is optimal. This means that, when entrepreneurs have little trust in the government's report of $y = h$, the inefficient government increases welfare by making a false report in state L. As the trust in the false report increases, welfare will start to decrease. The welfare results are summarized in the following proposition.

Proposition 5 *In state H, welfare is increasing in the entrepreneurs' trust in the public signal. In state L, the inefficient government can increase welfare by making false reports* *if the trust in the public signal is low. As the trust in the public signal grows, welfare will start to decrease.*

2.5. Conclusion

This paper analyzed the effects of short-term reputation concerns in the disclosure of public information in a coordination environment.

In equilibrium, when the efficient government is truthful, the inefficient government sends signals that are too optimistic, making false reports of a high productivity state with positive probability to be perceived as efficient. This creates a distortion in the entrepreneurs' beliefs about the productivity of investment. I find that false reports can increase welfare in the low productivity state. Following a false report, entrepreneurs overestimate the productivity of new venture and have more aggressive investment strategies. Since there is complementarity, entrepreneurs benefit from a higher level of aggregate investment. When agents distrust the government, the bias in the entrepreneurs' beliefs is small and welfare improving: the potential losses caused by overestimation of productivity are offset by the complementarity gains. As the trust in the false reports increases, there is too much investment and welfare starts to decrease. In the high productivity state however, welfare is increasing in the entrepreneurs' trust in the government. When the entrepreneurs do not trust a true report of a high productivity state, they underestimate the productivity of a new venture, there is less investment, and welfare is reduced.

There are two interesting extensions to the model: including a concern for welfare in the government's utility function; and introducing the concern for future reputation and the possibility of replacement. When welfare is taken into account, the efficient government might depart from a truthful policy to increase welfare in the low productivity state. If the government cares about the discounted value of being in office, it is possible to explore the tradeoff between current and future reputation. With the introduction of replacement, this framework can be used to analyze policy experiments concerning the frequency of elections.

For example, if the government wants to maximize its reputation every T periods, when elections are held, we can see how the choice of T affects welfare. We can also analyze how the strength of the government (or institutions) affects the incentives to disclose information. Suppose that whenever the reputation falls below a threshold $\underline{\mu}$, the incumbent is replaced, and the stronger the government, the lower is $\underline{\mu}$. In this case, weaker governments will place a higher weight on short-term reputation. This is equivalent to introducing the possibility of recall at every period.

APPENDIX

A.1. A numerical example

This section provides a numerical example for the baseline model with commitment. The state of fundamentals is uniformly distributed in $[0, 1]$:

$$\theta \sim U(0, 1).$$

The exchange rate in the absence of government intervention is

$$f(\theta) = \theta.$$

If the government maintains the peg, the exchange rate is

$$e^* = 1.$$

The cost of short selling is

$$t = 0.25.$$

The government's value of defending the currency is

$$v = 0.75.$$

The cost of defending the currency is

$$c(\alpha, \theta) = 1 - \theta + \alpha,$$

where α is the measure of speculators that attack the currency. [1]

For a given θ, the government decides to abandon the peg if the fraction α of speculators attacking the currency is at least $a(\theta)$, where

$$a(\theta) = \begin{cases} 0, & \text{if } \theta \in [0, 0.25] \\ \theta - 0.25, & \text{if } \theta \in (0.25, 1] \end{cases}.$$

Define $\underline{\theta}$ as the solution to $c(0, \theta) = v$, and define $\overline{\theta}$ as the solution to $e^* - f(\theta) = t$. We have

$$\underline{\theta} = 0.25,\ \overline{\theta} = 0.75.$$

Speculators receive a signal $x = \theta + \tilde{\varepsilon}$, where

$$\tilde{\varepsilon} \sim U(-\varepsilon, \varepsilon).$$

The precision of the signal is affected by the parameter ε:

$$\varepsilon = 0.1.$$

If a speculator receives the signal $x \in [-\varepsilon, 1 + \varepsilon]$, he will believe that θ is uniformly distributed in $[x - \varepsilon, x + \varepsilon] \cap [0, 1]$.

Let $\psi(k)$ solve

$$\frac{1}{2} - \frac{\psi(k)}{2\varepsilon} = k + \psi(k) - 0.25$$

[1]Note that $c(0,0) = 1 > v$ and $c(1,1) = 1 > v$, but $c(0.1, 0.8) = 1 + 0.1 - 0.8 = 0.3 < v$, so there are regions where the government decides to maintain the peg.

or

$$\psi(k)\left(1+\frac{1}{2\varepsilon}\right)=\frac{3}{4}-k$$

$$\psi(k)=\left(\frac{3}{4}-k\right)\left(1+\frac{1}{.2}\right)^{-1}=\left(\frac{3}{4}-k\right)(6)^{-1}=\frac{1}{8}-\frac{k}{6}.$$

The speculators payoff from following I_k when there is no public signal is given by[2]

$$\begin{aligned}u(k,I_k)=&\frac{1}{2\varepsilon}\int_{k-\varepsilon}^{k+\psi(k)}e^*-f(\theta)d\theta-t=5\int_{k-\varepsilon}^{k+\psi(k)}1-\theta d\theta-0.25.\\ =&\,5\left[\theta-\frac{\theta^2}{2}\right]\Bigg|_{k-\varepsilon}^{k+\psi(k)}-0.25\\ =&0.764k^2-1.854k+0.861.\end{aligned}$$

Proceeding numerically,

$$u(x^*,I_{x^*})=0 \iff x^*\tilde{=}0.626,$$

which implies

$$\theta^*\tilde{=}0.64.$$

A speculator will attack the currency $x<x^*$, and will not attack if $x>x^*$. Given this rule, the government will abandon the peg when $\theta<\theta^*$, and maintain the peg if $\theta>\theta^*$.

For a given θ, the fraction of speculators attacking the currency is

$$\mathbb{P}(\theta+\tilde{\varepsilon}<x^*)=\mathbb{P}(\tilde{\varepsilon}<x^*-\theta)=\begin{cases}0, & \text{if } \theta\geq x^*+\varepsilon\\ \frac{\varepsilon+x^*-\theta}{2\varepsilon}, & \text{if } \theta\in(x^*-\varepsilon,x^*+\varepsilon)\\ 1, & \text{if } \theta\leq x^*-\varepsilon\end{cases}$$

[2]For $k\in(0.1,0.9)$.

The expected payoff to the government is

$$\mathbb{P}(\theta < \theta^*) \cdot 0 + \mathbb{P}(\theta > \theta^*) \int_{\theta^*}^{1} v - (1 - \theta + \alpha(\theta))\, d\theta.$$

A.1.1. Public signal

Suppose that the government can commit to a public signal. The government chooses $m \in [0,1]$ and emits a signal

$$y(\theta) = \begin{cases} y_l, & \text{if } \theta \leq m \\ y_h, & \text{if } \theta > m \end{cases}.$$

Given the parameters above, $\underline{m} = x^* - \varepsilon = 0.526$. Figure 1 depicts the mass of speculators who attack the currency for different choices of m, compared to the case where there is no public signal. It also shows the critical mass of attackers needed for the government to abandon the peg, $a(\theta)$.

Suppose that, in the case of multiplicity given a choice of m, the government only cares about the lowest equilibrium payoff. In this case, the government chooses a partition P to maximize $V(P)$. From Theorem 3, the government wants to set m as close as possible to $\underline{m}$. Since $\underline{m} = x^* - \varepsilon$, from Theorem 2, for $m = \underline{m}$, there exists an equilibrium where speculators follow I_{x^*} when $y = y_h$. Hence, the choice $m = \underline{m}$ leads to multiple equilibria, and it is dominated by any $m \in (\underline{m}, \theta^*]$, which lead to a unique equilibrium. Theorem 3 thus implies that there is no equilibrium for this game. However, the government can still achieve a payoff arbitrarily close to $\bar{V} = \sup_P V(P) = \lim_{m \downarrow \underline{m}} V(P^m)$. Figure 2 depicts the government's payoff $V(P^m)$ for all possible choices of $m \in [0,1]$.

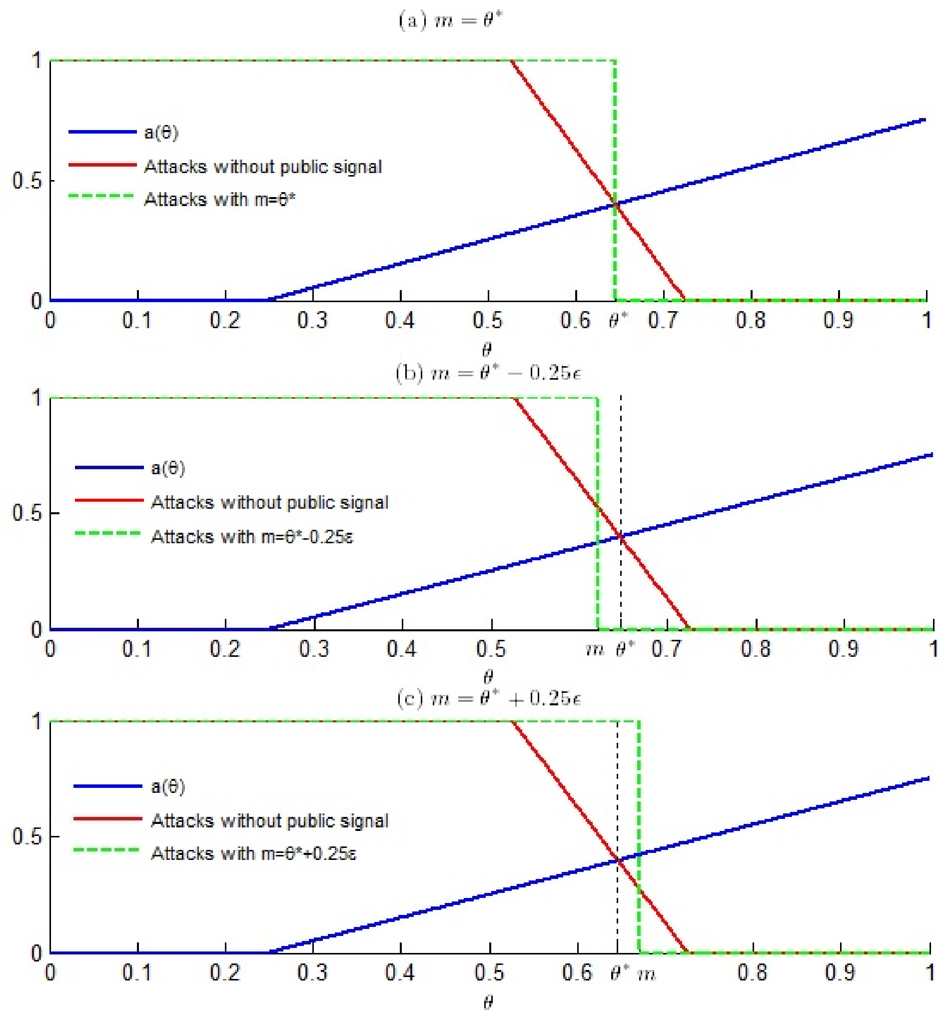

Figure 1: Mass of speculators attacking the currency for different choices of m, compared to the case with no public signal.

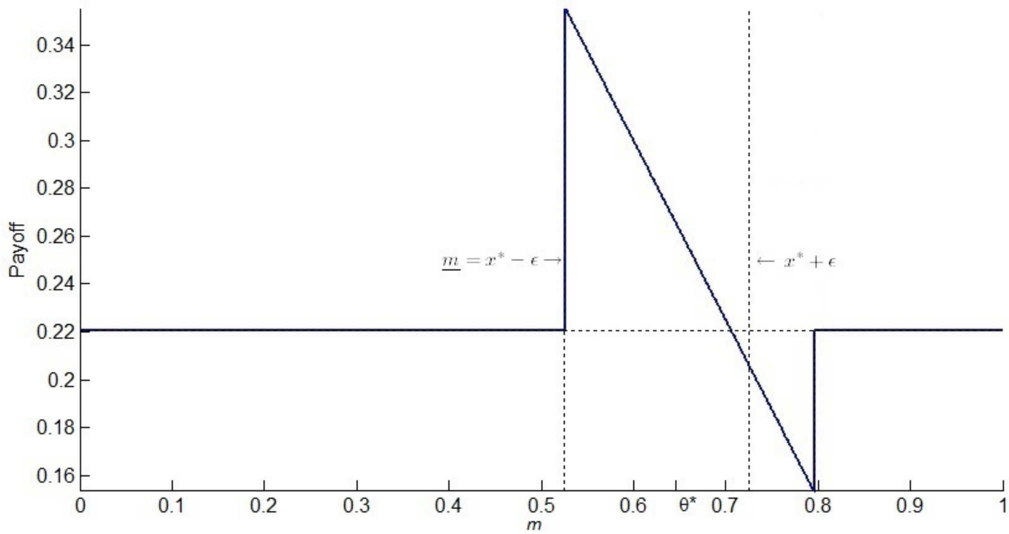

Figure 2: Government's payoff $V(P^m)$ given the choice of m.

A.2. Proofs for Chapter 1

A.2.1. Posteriors

For any pair of continuous random variables A and B, let g_{AB} denote their joint pdf. Let g_A and g_B denote the marginal pdfs, and let $g_{A|B}$ denote the pdf of A conditional on B. Finally, denote the cdfs by G_A and G_B. Following the main text, we denote the pdf of the idiosyncratic noise by g, and its cdf by G, ommiting the subscripts.

No public signal

For $x \in (-\varepsilon, 1+\varepsilon)$:

$$g_{\theta|x}(\theta|x) = \frac{g_{\theta x}(\theta, x)}{g_x(x)} = \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma}) g_\theta(\theta)}{\int_{-\infty}^{+\infty} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta}}$$
$$= \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x}{\sigma}\right) - G\left(\frac{x-1}{\sigma}\right)} \quad \text{, if } \theta \text{ is uniform on } [0,1].$$

For $x \in \{-\varepsilon, 1+\varepsilon\}$: $\mathbb{P}(\theta = 0 | x = -\varepsilon) = 1$; $\mathbb{P}(\theta = 1 | x = 1+\varepsilon) = 1$.

Public signal

Posterior of $\theta \in y_n$ conditional on the public signal $y = y_n$:

$$g_\theta(\theta|y_n) = \frac{g_\theta(\theta)}{P(m_{n-1} \le \theta \le m_n)} = \frac{g_\theta(\theta)}{G_\theta(m_n) - G_\theta(m_{n-1})}.$$

Distribution of x_i conditional on $y = y_n$:

$$\begin{aligned} P(x_i \le x|y_n) &= \frac{\int_{-\infty}^{x}\int_{m_{n-1}}^{m_n} g_{\theta x}(\theta, \tilde{x}) d\theta d\tilde{x}}{G_\theta(m_n) - G_\theta(m_{n-1})} = \frac{\int_{-\infty}^{x}\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g(\frac{\tilde{x}-\theta}{\sigma}) g_\theta(\theta) d\theta d\tilde{x}}{G_\theta(m_n) - G_\theta(m_{n-1})} \\ \Rightarrow g_x(x|y_n) &= \frac{\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g(\frac{x-\theta}{\sigma}) g_\theta(\theta) d\theta}{G_\theta(m_n) - G_\theta(m_{n-1})}. \end{aligned}$$

Hence, the posterior of θ conditional on (x, y_n) is

$$\begin{aligned} g_{\theta|x}(\theta|x; y_n) &= \frac{g_{\theta x}(\theta, x|y_n)}{g_x(x|y_n)} = \frac{g_{x|\theta}(x|\theta; y_n) g_\theta(\theta|y_n)}{g_x(x|y_n)} \\ &= \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma})[g_\theta(\theta)/(G_\theta(m_n) - G_\theta(m_{n-1}))]}{\left[\int_{m_{n-1}}^{m_1} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta}/(G_\theta(m_n) - G_\theta(m_{n-1}))\right]} \\ &= \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma}) g_\theta(\theta)}{\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta}} \\ &= \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma})}{G\left(\frac{x-m_{n-1}}{\sigma}\right) - G\left(\frac{x-m_n}{\sigma}\right)} \quad \text{, if } \theta \text{ is uniform on } [0,1]. \end{aligned}$$

Comparison

In the case of a two-element partition, when $y = y_l$ ($\theta \le m$):

$$g_{\theta|x}(\theta|x; y_l) = g_{\theta|x}(\theta|x) \gamma_{y_l}(x),$$

where $\gamma_{y_l}(x) = [\int_{-\infty}^{+\infty} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta} / \int_{-\infty}^{m} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta}] \geq 1$. And if $y = y_h$ ($\theta > m$):

$$g_{\theta|x}(\theta|x; y_h) = g_{\theta|x}(\theta|x)\gamma_{y_h}(x),$$

where $\gamma_{y_h}(x) = [\int_{-\infty}^{+\infty} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta} / \int_{m}^{+\infty} \frac{1}{\sigma} g(\frac{x-\tilde{\theta}}{\sigma}) g_\theta(\tilde{\theta}) d\tilde{\theta}] \geq 1$.

A.2.2. Proof of Lemma 1

Lemma 1. *For a given public signal y, if $\pi(x, y) \geq \pi'(x, y)$ for all x, then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x.*

Proof:

$$\pi(x, y) \geq \pi'(y, x) \forall x \Rightarrow s(\theta, \pi) \geq s(\theta, \pi') \Rightarrow A(\pi) \cap y \supseteq A(\pi') \cap y \Rightarrow u_y(x, \pi) \geq u_y(x, \pi').$$

□

A.2.3. Derivation of ψ

For $k \in [-\varepsilon, 1 + \varepsilon]$, define θ_k as

$$\theta_k = \sup\{\theta : s(\theta, I_k) \geq a(\theta)\}.$$

θ_k is the largest value of θ such that the government finds it optimal to abandon the peg when speculators' aggregate short sales are given by I_k. Since $s(\cdot, I_k)$ is decreasing and $a(\cdot)$ is increasing, the government abandons the peg if and only if $\theta \leq \theta_k$.

If $k \leq \underline{\theta} - \varepsilon$,

$$s(k + \varepsilon, I_k) = G\left(\frac{-\varepsilon}{\sigma}\right) = 0,$$

which implies that $\theta_k = \underline{\theta}$. If $k \in (\underline{\theta} - \varepsilon, 1 + \varepsilon]$,

$$s(k - \varepsilon, I_k) = G\left(\frac{\varepsilon}{\sigma}\right) = 1 > a(k - \varepsilon),$$

therefore θ_k is well-defined.[3] Note that θ_k is continuous in k.

Define $\bar{k}$ as the unique value of k that solves

$$G\left(\frac{k-1}{\sigma}\right) = a(1).$$

Hence $\bar{k} = 1 + \sigma G^{-1}(a(1))$. Since $a(1) \in (0,1)$, $\bar{k} \in (1 - \varepsilon, 1 + \varepsilon)$.

For $k \in (\underline{\theta} - \varepsilon, \bar{k}]$, θ_k is then the unique value of θ that solves

$$G\left(\frac{k-\theta}{\sigma}\right) = a(\theta). \tag{A.1}$$

If $\theta < \underline{\theta}$, the LHS of (A.1) is strictly positive, while the RHS equals 0, thus $\theta_k > \underline{\theta}$. Note that the LHS of (A.1) is strictly decreasing in θ, for $\theta \in (k - \varepsilon, k + \varepsilon)$, and constant otherwise. For $\theta > \underline{\theta}$, $a(\theta) \in (0,1)$, and it is strictly increasing. This implies that $\theta_k \in (k - \varepsilon, k + \varepsilon)$, and that θ_k is strictly increasing in k. Finally, if $k > \bar{k}$, $\theta_k = 1$.

Define the function ψ as $\psi(k) = \min\{\theta_k - k, \varepsilon\}$, for $k \in [-\varepsilon, 1 + \varepsilon]$. Thus

$$\psi(k) = \begin{cases} \varepsilon, & \text{if } k \leq \underline{\theta} - \varepsilon \\ -\sigma G^{-1}(a(\theta_k)) \in (-\varepsilon, \varepsilon), & \text{if } k \in (\underline{\theta} - \varepsilon, \bar{k}] \\ 1 - k \in [-\varepsilon, \varepsilon), & \text{if } k > \bar{k} \end{cases}.$$

From the continuity of θ_k, it follows that $\psi(k)$ is continuous. Since θ_k is strictly increasing for $k \in (\underline{\theta} - \varepsilon, \bar{k}]$, then $\psi(k)$ is strictly decreasing for $k > \underline{\theta} - \varepsilon$.

[3] $c(1,1) > v$ implies that $a(1) < 1$, thus $a(\theta) < 1$ for all θ.

A.2.4. Proof of Lemma 2

Lemma 2. *For a given public signal y, $u_y(k, I_k)$ is continuous in k, for all possible private signals $k \in X_y$.*

Proof: Using 1.11, the payoff function when $y = y_n$ is given by

$$u_{y_n}(k, I_k) = \int_{a_{y_n}}^{b_{y_n}} [e^* - f(\theta)]\phi_{y_n}(\theta|k)d\theta - t, \tag{A.2}$$

where $a_{y_n} = \max\{k - \varepsilon, m_{n-1}\}$, and $b_{y_n} = \max\{\min\{k + \psi(k), m_n\}, m_{n-1}\}$. Since $\phi_{y_n}(\cdot|k)$ and the limits of integration are continuous in k (because $\psi(\cdot)$ is continuous), $u_{y_n}(k, I_k)$ is continuous in k. □

A.2.5. Proof of Lemma 3

Lemma 3. *For $k \in (\varepsilon, 1 - \varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k.*

Proof: For $k \in (\varepsilon, 1 - \varepsilon)$

$$u(k, I_k) = \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta - t.$$

Differentiating $u(k, I_k)$ with respect to k and using the fact that $G\left(\frac{k-1}{\sigma}\right) = g\left(\frac{k-1}{\sigma}\right) = 0$, for $k < 1 - \varepsilon$, yield

$$\begin{aligned}
&\frac{d}{dk}u(k, I_k) \\
&= [e^* - f(k+\psi(k))](1+\psi'(k))\frac{\frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} - [e^* - f(k-\varepsilon)]\frac{\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} \\
&\qquad + \int_{k-\varepsilon}^{k+\psi(k)}[e^* - f(\theta)]\frac{\frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)}d\theta - \int_{k-\varepsilon}^{k+\psi(k)}[e^* - f(\theta)]\frac{1}{\sigma^2}\frac{g\left(\frac{k-\theta}{\sigma}\right)g\left(\frac{k}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)^2}d\theta \\
&\leq [e^* - f(k+\psi(k))]\frac{\frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} - [e^* - f(k-\varepsilon)]\frac{\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} \\
&\qquad + \int_{k-\varepsilon}^{k+\psi(k)}[e^* - f(\theta)]\frac{\frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)}d\theta,
\end{aligned}$$

where the inequality comes from $\psi'(k) \leq 0$, and from the fact that the last term on the RHS of the equality is positive. Define $\underline{\tilde{\varepsilon}}$ as

$$\underline{\tilde{\varepsilon}} = \inf\left\{\tilde{\varepsilon} \in \left[\frac{-\psi(k)}{\sigma}, \frac{\varepsilon}{\sigma}\right] : g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} > \tilde{\varepsilon}\right\}.$$

From (1.1), $\underline{\tilde{\varepsilon}}$ is well defined. Furthermore, $g'(\tilde{\varepsilon}) \geq 0$, for $\tilde{\varepsilon} \leq \underline{\tilde{\varepsilon}}$, and $g'(\tilde{\varepsilon}) \leq 0$, for $\tilde{\varepsilon} > \underline{\tilde{\varepsilon}}$. Define $\underline{\tilde{\theta}}$ as

$$\underline{\tilde{\theta}} = k - \sigma\underline{\tilde{\varepsilon}}.$$

Hence $\underline{\tilde{\theta}} \in [k-\varepsilon, k+\psi(k)]$. We then have

$$
\begin{aligned}
&G\left(\frac{k}{\sigma}\right)\left[\frac{d}{dk}u(k, I_k)\right] \\
&\leq [e^* - f(k+\psi(k))]\frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right) - [e^* - f(k-\varepsilon)]\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right) \\
&\qquad + \int_{k-\varepsilon}^{\min\{\underline{\tilde{\theta}}, k+\psi(k)\}} [e^* - f(\theta)]\frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&\qquad + \int_{\min\{\underline{\tilde{\theta}}, k+\psi(k)\}}^{k+\psi(k)} [e^* - f(\theta)]\frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&\leq [e^* - f(k+\psi(k))]\frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right) - [e^* - f(k-\varepsilon)]\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right) \\
&\qquad + [e^* - f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\})]\int_{k-\varepsilon}^{\min\{\underline{\tilde{\theta}}, k+\psi(k)\}} \frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&\qquad + [e^* - f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\})]\int_{\min\{\underline{\tilde{\theta}}, k+\psi(k)\}}^{k+\psi(k)} \frac{1}{\sigma^2}g'\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&= [e^* - f(k+\psi(k))]\frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right) - [e^* - f(k-\varepsilon)]\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right) \\
&\qquad + [e^* - f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\})]\left[\frac{1}{\sigma}g\left(\frac{\varepsilon}{\sigma}\right) - \frac{1}{\sigma}g\left(\frac{k-\min\{\underline{\tilde{\theta}}, k+\psi(k)\}}{\sigma}\right)\right] \\
&\qquad + [e^* - f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\})]\left[\frac{1}{\sigma}g\left(\frac{k-\min\{\underline{\tilde{\theta}}, k+\psi(k)\}}{\sigma}\right) - \frac{1}{\sigma}g\left(\frac{-\psi(k)}{\sigma}\right)\right] \\
&= \frac{1}{\sigma}\left\{g\left(\frac{-\psi(k)}{\sigma}\right)\left[f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\}) - f(k+\psi(k))\})\right]\right. \\
&\qquad\qquad\qquad\qquad \left. + g\left(\frac{\varepsilon}{\sigma}\right)\left[f(k-\varepsilon) - f(\min\{\underline{\tilde{\theta}}, k+\psi(k)\})\right]\right\}, \\
&< 0,
\end{aligned}
$$

which implies that $u(k, I_k)$ is strictly increasing in k. □

A.2.6. Proof that Assumption 1 holds for concave or normal distributions

Here we show two assumptions on the distribution of the idiosyncratic noise that guarantee that Assumption 1 is satisfied. The first assumption is that the probability density function g is concave on $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The second assumption is that the noise follows a truncated normal distribution on $[-\bar{\varepsilon}, \bar{\varepsilon}]$.

Without loss of generality, assume that $\sigma = 1$. In this case, $\varepsilon = \bar{\varepsilon}$. Let the public signal be y and consider two private signals x_1 and x_2, with $x_1 < x_2$. There are five possible cases:

1. $\theta \leq x_1 - \varepsilon$: $\Phi_y(\theta|x_1) = \Phi_y(\theta|x_2) = 0$;
2. $\theta \geq x_2 + \varepsilon$: $\Phi_y(\theta|x_1) = \Phi_y(\theta|x_2) = 1$;
3. $\theta \in (x_1 - \varepsilon, x_2 - \varepsilon)$: $\Phi_y(\theta|x_1) > \Phi_y(\theta|x_2) = 0$;
4. $\theta \in (x_1 + \varepsilon, x_2 + \varepsilon)$: $1 = \Phi_y(\theta|x_1) > \Phi_y(\theta|x_2)$;
5. $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.

To prove that Assumption 1 is satisfied, it is left to show that $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for all $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$. Note that, in this case, $x_2 < x_1 + 2\varepsilon$.

Concave distribution

Let the probability density function of the idiosyncratic noise, g, be concave on $[-\varepsilon, \varepsilon]$. The density of θ conditional on a public signal $y = y_n$ and a private signal x is given by

$$\phi_y(\theta|x) = \frac{g(x-\theta)}{G(x - m_{n-1}) - G(x - m_n)}.$$

Consider two private signals x_1 and x_2, with $x_1 < x_2$ and $x_2 < x_1 + 2\varepsilon$, and define $\delta = x_2 - x_1 < 2\varepsilon$. For $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$

$$\frac{\phi_y(\theta|x_1)}{\phi_y(\theta|x_2)} = \frac{g(x_1 - \theta)}{g(x_1 - \theta + \delta)}\bar{c},$$

where $\bar{c} = [G(x_2 - m_{n-1}) - G(x_2 - m_n)]/[G(x_1 - m_{n-1}) - G(x_1 - m_n)]$.

To prove that Assumption 1 holds for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, two results are needed.

Claim 1. *If g is concave on $[-\varepsilon, \varepsilon]$, then $\phi_y(\cdot|x_1)$ crosses $\phi_y(\cdot|x_1)$ at most once for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.*

Proof: Suppose that there exists θ_1 and θ_2 in $[x_2 - \varepsilon, x_1 + \varepsilon]$, with $\theta_1 < \theta_2$, such that

$$\frac{\phi_y(\theta_1|x_1)}{\phi_{y_n}(\theta_1|x_2)} = \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_2|x_2)} = 1.$$

Define $\varepsilon_H = x_1 - \theta_1$ and $\varepsilon_L = x_1 - \theta_2$. Hence

$$\bar{c}g(\varepsilon_H) = g(\varepsilon_H + \delta),$$
$$\bar{c}g(\varepsilon_L) = g(\varepsilon_L + \delta).$$

There are three cases:

1. $\bar{c} > 1$:

$$g(\varepsilon_H) < g(\varepsilon_H + \delta),$$
$$g(\varepsilon_L) < g(\varepsilon_L + \delta).$$

It must be true that $g(\varepsilon_H) > g(\varepsilon_L)$, otherwise g would decrease or be constant somewhere between ε_L and ε_H, and then increase somewhere between ε_H and $\varepsilon_H + \delta$, a contradiction with the concavity of g. The slope of the line segment that connects points $(\varepsilon_L, g(\varepsilon_L))$ and $(\varepsilon_L + \delta, g(\varepsilon_L + \delta))$ is given by

$$S_L = (\bar{c} - 1)\frac{g(\varepsilon_L)}{\delta},$$

and the slope of the line segment that connects $(\varepsilon_H, g(\varepsilon_H))$ and $(\varepsilon_H + \delta, g(\varepsilon_H + \delta))$ is given by

$$S_H = (\bar{c} - 1)\frac{g(\varepsilon_H)}{\delta}.$$

Since $g(\varepsilon_H) > g(\varepsilon_L)$, it follows that $S_H > S_L$, a contradiction with the concavity of g.

2. $\bar{c} < 1$:

$$g(\varepsilon_H) > g(\varepsilon_H + \delta),$$
$$g(\varepsilon_L) > g(\varepsilon_L + \delta).$$

It must be true that $g(\varepsilon_L) > g(\varepsilon_H)$, otherwise $g(\varepsilon_L) \leq g(\varepsilon_H)$ and $g(\varepsilon_L + \delta) = \bar{c}g(\varepsilon_L) \leq \bar{c}g(\varepsilon_H) = g(\varepsilon_H + \delta)$. In this case, g would decrease between ε_L and $\varepsilon_L + \delta$, and then increase or be constant somewhere between $\varepsilon_L + \delta$ and $\varepsilon_H + \delta$, a contradiction with the concavity of g. The slope of the line segment that connects points $(\varepsilon_L, g(\varepsilon_L))$ and $(\varepsilon_L + \delta, g(\varepsilon_L + \delta))$ is given by

$$S_L = (\bar{c} - 1)\frac{g(\varepsilon_L)}{\delta},$$

and the slope of the line segment that connects $(\varepsilon_H, g(\varepsilon_H))$ and $(\varepsilon_H + \delta, g(\varepsilon_H + \delta))$ is given by

$$S_H = (\bar{c} - 1)\frac{g(\varepsilon_H)}{\delta}.$$

Since $g(\varepsilon_L) > g(\varepsilon_H)$ and $\bar{c} < 1$, it follows that $S_H > S_L$, a contradiction with the concavity of g.

3. $c = 1$:

$$g(\varepsilon_H) = g(\varepsilon_H + \delta),$$
$$g(\varepsilon_L) = g(\varepsilon_L + \delta),$$

thus it must be the case that $g(\varepsilon_H) = g(\varepsilon_L)$ and g is flat between ε_L and $\varepsilon_H + \delta$.

$\square$

Claim 2. *If g is concave on $[-\varepsilon, \varepsilon]$, then $\phi_y(\cdot|x_2)$ can only cross $\phi_y(\cdot|x_1)$ from below for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.*

Proof: Suppose that $\phi_y(\theta|x_2)$ crosses $\phi_y(\theta|x_1)$ from above in $[x_2-\varepsilon, x_1+\varepsilon]$. Then, there exist θ_1 and θ_2 in $[x_2-\varepsilon, x_1+\varepsilon]$, with $\theta_1<\theta_2$, such that:

$$\phi_y(\theta_1|x_2) = \phi_y(\theta_1|x_1),$$
$$\phi_y(\theta_2|x_2) < \phi_y(\theta_2|x_1).$$

Define $\varepsilon_H = x_1-\theta_1$ and $\varepsilon_L = x_1-\theta_2$. The inequalities above imply

$$\bar{c}g(\varepsilon_H) = g(\varepsilon_H+\delta),$$
$$\bar{c}g(\varepsilon_L) > g(\varepsilon_L+\delta).$$

Denote the slope of the line segment that connects points $(\varepsilon_I, g(\varepsilon_I))$ and $(\varepsilon_I+\delta, g(\varepsilon_I+\delta))$ by S_I, for $I\in\{L,H\}$. There are three cases:

1. $\bar{c}=1$: then $S_L<0$ and $S_H, 0$, a contradiction with the concavity of g.

2. $\bar{c}>1$: then $g(\varepsilon_H)<g(\varepsilon_H+\delta)$. It follows that $g(\varepsilon_L)\le g(\varepsilon_H)$, otherwise g would decrease between ε_L and ε_H, and then increase between ε_H and $\varepsilon_H+\delta$, a contradiction with the concavity of g. Thus

$$S_L < (\bar{c}-1)\frac{g(\varepsilon_L)}{\delta} \le (\bar{c}-1)\frac{g(\varepsilon_H)}{\delta} = S_H,$$

   a contradiction with the concavity of g.

3. $\bar{c}<1$: then $g(\varepsilon_L)>g(\varepsilon_L+\delta)$. It follows that $g(\varepsilon_L)>g(\varepsilon_H)$, otherwise

$$g(\varepsilon_H+\delta)=\bar{c}g(\varepsilon_H)\ge\bar{c}g(\varepsilon_L)>g(\varepsilon_L+\delta),$$

   therefore g would decrease between ε_L and $\varepsilon_L+\delta$, and then increase between $\varepsilon_L+\delta$

and $\varepsilon_H + \delta$, a contradiction with the concavity of g. Thus

$$S_L < -(1-\bar{c})\frac{g(\varepsilon_L)}{\delta} \leq -(1-\bar{c})\frac{g(\varepsilon_H)}{\delta} = S_H,$$

a contradiction with the concavity of g.

$\square$

Claims 1 and 2 imply that, if there exists $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, such that $\phi_y(\theta|x_2) > \phi_y(\theta|x_1)$, then $\phi_y(\tilde{\theta}|x_2) \geq \phi_y(\tilde{\theta}|x_1)$ for all $\tilde{\theta} > \theta$. This implies that, if there exists $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, such that $\Phi_y(\theta|x_2) > \Phi_y(\theta|x_1)$, then $\Phi_y(\tilde{\theta}|x_2) > \Phi_y(\tilde{\theta}|x_1)$, for all $\tilde{\theta} > \theta$. In particular, $\Phi_y(x_1 + \varepsilon|x_2) > \Phi_y(x_1 + \varepsilon|x_1) = 1$, a contradiction. Thus $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$, for all $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, and Assumption 1 holds.

Normal distribution

Suppose that the idiosyncratic noise follows a truncated normal distribution on $[-\varepsilon, \varepsilon]$, with the originating distribution having mean μ and variance ν^2. Let the public signal be y and consider two private signals x_1 and x_2, with $x_1 < x_2$ and $x_2 < x_1 + 2\varepsilon$. To prove that $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, it suffices to show that the following monotone likelihood ratio holds

$$\frac{\phi_y(\theta_2|x_2)}{\phi_y(\theta_1|x_2)} \geq \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_1|x_1)}, \quad \forall \theta_1 < \theta_2.$$

Let $\theta_1, \theta_2 \in [x_2 - \varepsilon, x_1 + \varepsilon]$. Then

$$
\begin{aligned}
\frac{\phi_y(\theta_2|x_2)}{\phi_y(\theta_1|x_2)} &\geq \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_1|x_1)} \\
\Leftrightarrow \frac{\exp\left(-\frac{(x_2-\theta_2-\mu)^2}{2\nu^2}\right)}{\exp\left(\frac{-(x_2-\theta_1-\mu)^2}{2\nu^2}\right)} &\geq \frac{\exp\left(-\frac{(x_1-\theta_2-\mu)^2}{2\nu^2}\right)}{\exp\left(-\frac{(x_1-\theta_1-\mu)^2}{2\nu^2}\right)} \\
\Leftrightarrow \exp\left(\frac{-(x_2-\theta_2-\mu)^2+(x_2-\theta_1-\mu)^2}{2\nu^2}\right) &\geq \exp\left(\frac{-(x_1-\theta_2-\mu)^2+(x_1-\theta_1-\mu)^2}{2\nu^2}\right) \\
\Leftrightarrow -(x_2-\theta_2-\mu)^2+(x_2-\theta_1-\mu)^2 &\geq -(x_1-\theta_2-\mu)^2+(x_1-\theta_1-\mu)^2 \\
\Leftrightarrow -(x_2-\theta_2)^2+2\mu(\theta_1-\theta_2)+(x_2-\theta_1)^2 &\geq -(x_1-\theta_2)^2+2\mu(\theta_1-\theta_2)+(x_1-\theta_1)^2 \\
\Leftrightarrow -x_2^2+2x_2\theta_2-\theta_2^2+x_2^2-2x_2\theta_1+\theta_1^2 &\geq -x_1^2+2x_1\theta_2-\theta_2^2+x_1^2-2x_1\theta_1+\theta_1^2 \\
\Leftrightarrow 2x_2(\theta_2-\theta_1) &\geq 2x_1(\theta_2-\theta_1) \\
\Leftrightarrow \theta_1 &\leq \theta_2,
\end{aligned}
$$

which completes the proof.

A.2.7. Lemma 9

Lemma 15 *Suppose that $y = y_n$ and that speculators follow I_k, for $k \in X_{y_n}$. When a speculator receives the private signal $x = k$, the payoff from attacking, $u_{y_n}(k, I_k)$, is continuous in both m_{n-1} and m_n. The payoff $u_{y_n}(k, I_k)$ is decreasing in m_{n-1} for $k < m_{n-1}+\varepsilon$, and constant otherwise. It is also decreasing in m_n for $k > m_n-\varepsilon$, and constant otherwise.*

Proof: Without loss in generality, let $y = y_2$. Then,

$$
u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{\max\{\min\{k+\psi(k), m_2\}, m_1\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,
$$

where

$$
D(k, m_1, m_2) = G\left(\frac{k - m_1}{\sigma}\right) - G\left(\frac{k - m_2}{\sigma}\right).
$$

The limits of integration are continuous in m_{n-1} and m_n, and, since G is a continuous function, D is continuous in all of its arguments. Hence $u_{y_2}(k, I_k)$ is continuous in m_{n-1} and m_n.

If $m_1 - \psi(k) \geq k$, then $u_{y_2}(k, I_k) = -t$, which is constant in m_1. If $k > m_1 + \varepsilon$ then the limits of integration above are constant in m_1 and so is $D(k, m_1, m_2)$,[4] therefore $u_{y_2}(k, I_k)$ is constant in m_1. Now consider the case $m_1 - \psi(k) < k \leq m_1 + \varepsilon$:

$$u_{y_2}(k, I_k) = \int_{m_1}^{\min\{k+\psi(k), m_2\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

then

$$\begin{aligned}
\frac{\partial}{\partial m_1} u_{y_2}(k, I_k) &= -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \\
&\quad + \left[\frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)}\right] \int_{m_1}^{\min\{k+\psi(k), m_2\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \\
&< -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \int_{m_1}^{\min\{k+\psi(k), m_2\}} \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta\right] \\
&= -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \frac{G\left(\frac{k-m_1}{\sigma}\right) - G\left(\frac{k-\min\{k+\psi(k), m_2\}}{\sigma}\right)}{D(k, m_1, m_2)}\right] \\
&\leq 0,
\end{aligned}$$

which completes the proof for m_1.

Let $k \leq m_2 - \varepsilon$. Then

$$u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{\max\{k+\psi(k), m_1\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

Since the limits of integration above are constant in m_2, and so is $D(k, m_1, m_2)$,[5] then $u_{y_2}(k, I_k)$ is constant in m_2. The same is true for $k \leq m_2 - \psi(k)$.

[4]For all m_1 such that $m_1 < k - \varepsilon$: $\frac{k-m_1}{\sigma} > \bar{\varepsilon} \Rightarrow G(\frac{k-m_1}{\sigma}) = 1$

[5]For all m_2 such that $m_2 \geq k + \varepsilon$: $\frac{k-m_2}{\sigma} \leq -\bar{\varepsilon} \Rightarrow G(\frac{k-m_2}{\sigma}) = 0$

Now suppose $m_2 < k + \psi(k)$. The payoff becomes

$$u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{m_2} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

then

$$\begin{aligned}
\frac{\partial}{\partial m_2} u_{y_2}(k, I_k) &= [e^* - f(m_2)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \\
&\quad - \left[\frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)}\right] \int_{\max\{m_1, k-\varepsilon\}}^{m_2} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \\
&< [e^* - f(m_2)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \int_{\max\{m_1, k-\varepsilon\}}^{m_2} \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta\right] \\
&= [e^* - f(m_2)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \frac{G\left(\frac{k-\max\{m_1, k-\varepsilon\}}{\sigma}\right) - G\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)}\right] \\
&= 0.
\end{aligned}$$

The last inequality comes from the fact that

$$\begin{aligned}
G\left(\frac{k - \max\{m_1, k-\varepsilon\}}{\sigma}\right) &= \min\left\{G\left(\frac{k-m_1}{\sigma}\right), G\left(\frac{\varepsilon}{\sigma}\right)\right\} = \min\left\{G\left(\frac{k-m_1}{\sigma}\right), 1\right\} \\
&= G\left(\frac{k-m_1}{\sigma}\right) \\
\Rightarrow G\left(\frac{k - \max\{m_1, k-\varepsilon\}}{\sigma}\right) - G\left(\frac{k-2}{\sigma}\right) &= D(k, m_1, m_2).
\end{aligned}$$

$\square$

A.2.8. Proof of Theorem 2

First, we need to compare $u_y(k, I_k)$ and $u(k, I_k)$. Note that for $k \leq m - \varepsilon$, from (1.12) and (A.2), the limits of integration are[6]

$$a_{y_l} = a \quad , \quad b_{y_l} = b,$$

and the density functions are the same, which implies that $u_{y_l}(k, I_k)$ equals the payoff function $u(k, I_k)$. For $k > m + \varepsilon$, from (1.12) and (A.2),

$$a_{y_h} = a \quad , \quad b_{y_h} = b,$$

and the density functions are the same, which implies that $u_{y_h}(k, I_k)$ equals the payoff function $u(k, I_k)$. From Lemma 2, the continuity of $u_{y_h}(k, I_k)$ in k implies that $u_{y_h}(k, I_k) = u(k, I_k)$ for $k = m+\varepsilon$. The comparison between $u_y(k, I_k)$ and $u(k, I_k)$ when $k \in (m-\varepsilon, m+\varepsilon)$ is analyzed in the two following lemmas.

Lemma 16 *If the public signal is $y = y_l$, then $u_{y_l}(k, I_k) > u(k, I_k)$ for all $k \in (m-\varepsilon, m+\varepsilon)$.*

Proof:

$$\begin{aligned}
& u_{y_l}(k, I_k) - u(k, I_k) \\
&= \int_{\max\{k-\varepsilon,0\}}^{\min\{k+\psi(k),m\}} [e^* - f(\theta)]\phi_{y_l}(\theta|k)d\theta - \int_{\max\{k-\varepsilon,0\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta \\
&= \left(\frac{1}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} - \frac{1}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} \right) \int_{\max\{k-\varepsilon,0\}}^{\min\{k+\psi(k),m\}} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&\quad - \int_{\min\{k+\psi(k),m\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta
\end{aligned}$$

[6] Here $y_l = y_1$, $y_h = y_2$, and $m = m_1$.

If $k+\psi(k) \leq m$, the last integral equals zero, therefore $u_{y_l}(k, I_k) > u(k, I_k)$. For $k+\psi(k) > m$,

$$
\begin{aligned}
&[u_{y_l}(k, I_k) - u(k, I_k)] \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)\right] \\
=&\left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}\right) \int_{\max\{k-\varepsilon,0\}}^{m} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&- \int_{m}^{k+\psi(k)} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
>&[e^* - f(m)] \left[\left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}\right) \int_{\max\{k-\varepsilon,0\}}^{m} \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \right. \\
&\qquad\qquad\qquad\qquad \left. - \int_{m}^{k+\psi(k)} \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \right] \\
=&[e^* - f(m)] \left\{\left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}\right) \left[G\left(\frac{k-\max\{k-\varepsilon,0\}}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)\right] \right. \\
&\qquad\qquad \left. - \left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right)\right]\right\} \\
=&[e^* - f(m)] \left\{\left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}\right) \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)\right] \right. \\
&\qquad\qquad \left. - \left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right)\right]\right\} \\
=&[e^* - f(m)] \left[G\left(\frac{-\psi(k)}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)\right] \\
\geq& 0,
\end{aligned}
$$

where the last inequality comes from $k+\psi(k) \leq 1$. This implies that $u_{y_l}(k, I_k) > u(k, I_k)$, which completes the proof. □

Lemma 17 *If the public signal is $y = y_h$, then $u_{y_h}(k, I_k) < u(k, I_k)$ for all $k \in (m-\varepsilon, m+\varepsilon)$.*

Proof: First note that if $k+\psi(k) \leq m$, $u_{y_h}(k, I_k) = -t < u(k, I_k)$. If $k+\psi(k) > m$,

$$
\begin{aligned}
&[u(k,I_k) - u_{y_h}(k,I_k)]\left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)\right] \\
=&\int_{\max\{k-\varepsilon,0\}}^{k+\psi(k)} [e^* - f(\theta)]\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&-\left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)\right]\int_m^{k+\psi(k)}[e^* - f(\theta)]\phi_{y_h}(\theta|k)d\theta \\
=&\int_{\max\{k-\varepsilon,0\}}^{m} [e^* - f(\theta)]\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)d\theta \\
&-\left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}\right)\int_m^{k+\psi(k)}[e^* - f(\theta)]\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)d\theta \\
<&[e^* - f(m)]\left[\int_{\max\{k-\varepsilon,0\}}^{m}\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)d\theta\right. \\
&\qquad\left.-\left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}\right)\int_m^{k+\psi(k)}\frac{1}{\sigma}g\left(\frac{k-\theta}{\sigma}\right)d\theta\right] \\
=&[e^* - f(m)]\left\{G\left(\frac{k-\max\{k-\varepsilon,0\}}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)\right. \\
&\qquad\left.-\left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}\right)\left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right)\right]\right\} \\
\leq&[e^* - f(m)]\left[G\left(\frac{k-\max\{k-\varepsilon,0\}}{\sigma}\right) - G\left(\frac{k}{\sigma}\right)\right] \\
=&0,
\end{aligned}
$$

which implies that $u(k, I_k) < u_{y_h}(k, I_k)$, therefore the proof is complete. □

Now we can prove Theorem 2:

Theorem 2. *Fix m. If $m = \theta^*$, there is a unique equilibrium, in which speculators follow the public signal. If $m \neq \theta^*$, the equilibrium may not be unique. There are bounds $\underline{x}^* \geq x^*$ and $\bar{x}^* \leq x^*$ such that, in any equilibrium, $\pi(x, y_l) \geq I_{\underline{x}^*}(x)$ and $\pi(x, y_h) \leq I_{\bar{x}^*}(x)$ for all x. The equilibria are as follows:*

i. if $m < \theta^*$: speculators always attack the currency and the peg is abandoned if $y = y_l$;

moreover, if $m \in (x^* - \varepsilon, \theta^*)$*, then* $\bar{x}^* < x^*$*;*

ii. *if* $m > \theta^*$*: the currency is not attacked and the peg defended if* $y = y_h$*; moreover, if* $m \in (\theta^*, x^* + \varepsilon)$*, then* $\underline{x}^* > x^*$*.*

Proof: Now consider any possible strategy profile for the speculators. For $y \in \{y_l, y_h\}$, let $\pi(x, y)$ denote the proportion of speculators who attack the currency given a private signal x. Define $\underline{x}_y$ and $\bar{x}_y$ as

$$\underline{x}_y = \inf\{x \in X_y : \pi(x, y) < 1\}, \qquad \text{and} \qquad \bar{x}_y = \sup\{x \in X_y : \pi(x, y) > 0\}.$$

Note that $\underline{x}_y \leq \bar{x}_y$. If $\underline{x}_y \in X_y$, then from Lemma 1

$$u_y(\underline{x}_y, I_{\underline{x}_y}) \leq u_y(\underline{x}_y, \pi) \leq 0, \tag{A.3}$$

and if $\bar{x}_y \in X_y$

$$u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0. \tag{A.4}$$

Using Lemma 3, the proof of existence and uniqueness of equilibrium in the game without public signal is analogous to the one in Morris and Shin (1998). The speculators follow a cutoff strategy I_{x^*}, such that $u(x^*, I_{x^*}) = 0$, with $x^* \in (\varepsilon, 1 - \varepsilon)$. Since $u(k, I_k) > 0$ for $k \leq \varepsilon$, and $u(k, I_k) < 0$ for $k \geq 1 - \varepsilon$, it follows from Lemma 3 that $u(k, I_k) > 0$ for $k < x^*$, and that $u(k, I_k) < 0$ for $k > x^*$.

First, let $\theta \in y_l = [0, m]$. From Lemma 16, $u_{y_l}(k, I_k) \geq u(k, I_k)$, with strict inequality for $k \in (m - \varepsilon, m + \varepsilon)$. If $x^* \notin X_{y_l} = [-\varepsilon, m + \varepsilon]$, then $u_{y_l}(k, I_k)$ is strictly positive for all k. From (A.3) all speculators attack the currency for $\theta \in y_l$, therefore the peg is always abandoned. If $x^* \in X_{y_l}$, then

$$u_{y_l}(x^*, I_{x^*}) \geq u(x^*, I_{x^*}) = 0,$$

with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. Hence, either every speculator attacks for all $x \in X_{y_l}$, in which case $\underline{\theta}^* = m$, or $\underline{x}_{y_l} \geq x^*$, with strict inequality if $m \in (x^* - \varepsilon, x^* + \varepsilon)$. In the latter case, all speculators attack the curency for $x < \underline{x}_{y_l}$. This guarantees the existence of $\underline{\theta}^* \in [\theta^*, m]$ such that the government always abandons the currency peg for all $\theta \leq \underline{\theta}^*$.[7] Furthermore, if $m \in (x^* - \varepsilon, x^* + \varepsilon)$ then $\underline{\theta}^* > \theta^*$.

Now let $\theta \in y_h = (m, 1]$. From Lemma 17, $u_{y_h}(k, I_k) \leq u(k, I_k)$, with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. If $x^* \notin X_{y_h} = (m - \varepsilon, 1 + \varepsilon]$, then $u_{y_h}(k, I_k)$ is strictly negative for all k. From (A.4) the currency is not attacked for any $\theta \in y_h$, and the government finds it optimal to keep the peg. If $x^* \in X_{y_h}$, then

$$u_{y_h}(x^*, I_{x^*}) \leq u(x^*, I_{x^*}) = 0,$$

with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. Hence, either $\bar{x}_{y_h} \leq x^*$, with strict inequality if $m \in (x^* - \varepsilon, x^* + \varepsilon)$, or the currency is never attacked, in which case $\bar{\theta}^* = m$. In the former case, no speculator attacks the curency for $x > \bar{x}_{y_h}$, therefore there exists $\bar{\theta}^* \in [m, \theta^*]$ such that the currency peg is never abandoned for $\theta > \bar{\theta}^*$.[8] Furthermore, if $m \in (x^* - \varepsilon, x^* + \varepsilon)$ then $\bar{\theta}^* < \theta^*$.

For $m = \theta^*$, since $\underline{\theta}^* \in [\theta^*, m]$ and $\bar{\theta}^* \in [m, \theta^*]$, it must be the case that $\underline{\theta}^* = \bar{\theta}^* = m$. Thus the unique equilibrium involves coordination on the public signal.

For $m < \theta^*$, then the currency is always attacked on $y_l = [0, m]$ and $\underline{\theta}^* = m$, otherwise $\theta^* \leq \underline{\theta}^* \leq m$, a contradiction. Since $m < \theta^*$, then $x^* \in X_{y_h}$. Hence, if $x^* \notin (m - \varepsilon, m + \varepsilon)$, then $\bar{\theta}^* \in [m, \theta^*]$, and if $x^* \in (m - \varepsilon, m + \varepsilon)$, then $\bar{\theta}^* \in [m, \theta^*)$. In the latter case, $\bar{x}_{y_h} < x^*$.

For $m > \theta^*$, then the currency is never attacked on $y_h = (m, 1]$ and $\bar{\theta}^* = m$, otherwise $m \leq \bar{\theta}^* \leq \theta^*$, a contradiction. Since $m > \theta^*$, then $x^* \in X_{y_l}$. Hence, if $x^* \notin (m - \varepsilon, m + \varepsilon)$,

[7] Take for example $\underline{\theta}^* = \theta_{\underline{x}_{y_l}}$, the value of θ that makes the government indifferent when speculators follow $I_{\underline{x}_{y_l}}$ $(s(\theta_{\underline{x}_{y_l}}, I_{\underline{x}_{y_l}}) = a(\theta_{\underline{x}_{y_l}}))$.

[8] Take for example $\bar{\theta}^* = \theta_{\bar{x}_{y_h}}$, the value of θ that makes the government indifferent when speculators follow $I_{\bar{x}_{y_h}}$ $(s(\theta_{\bar{x}_{y_h}}, I_{\bar{x}_{y_h}}) = a(\theta_{\bar{x}_{y_h}}))$.

then $\underline{\theta}^* \in [\theta^*, m]$, and if $x^* \in (m-\varepsilon, m+\varepsilon)$, then $\underline{\theta}^* \in (\theta^*, m]$. In the latter case, $\underline{x}_{y_l} > x^*$.

A.2.9. Lemma 12

Lemma 18 *Suppose that Assumption 1 is satisfied. Then,* $\underline{m} < \bar{\theta}$.

Proof: We need to find $m < \bar{\theta}$ such that $u_{(m,1]}(k, I_k) < 0$ for all k. Consider the partition $P^{\bar{\theta}}$ and let $\bar{k}$ solve $\theta_{\bar{k}} = \bar{\theta}$.[9]

We claim that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta < 0$ for all $k \in (\bar{\theta} - \varepsilon, 1+\varepsilon)$. To see this, let $k \leq \bar{k}$. If speculators follow I_k, then the threshold for the government to abandon the peg is $\theta_k \leq \bar{\theta}$, which means that the government does not abandon the peg on $(\bar{\theta}, 1]$. Hence $u_{(\bar{\theta},1]}(k, I_k) = -t$ for any $k \leq \bar{k}$. For $k > \bar{k}$

$$u_{(\bar{\theta},1]}(k, I_k) \leq u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq u_{(\bar{\theta},1]}(\bar{k}, I_{1+\varepsilon}) \equiv \delta < 0,$$

where the first inequality comes from Lemma 1, the second inequality comes from Lemma 4, and the last inequality comes from the fact that it is never profitable to attack when $y = (\bar{\theta}, 1]$. Since $\delta \geq -t$, we have that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta$ for all k.

Define l_m^1 and l_m^2 as

$$l_m^1 = \lim_{k \downarrow \bar{k}} u_{(m,1]}(k, I_{1+\varepsilon}),$$

and

$$l_m^2 = \lim_{k \downarrow \bar{\theta} - \varepsilon} u_{(m,1]}(k, I_{\bar{k}}).$$

Since $u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq \delta$ for all $k > \bar{k}$, continuity implies that $l_{\bar{\theta}}^1 \leq \delta$. Since $u_{(\bar{\theta},1]}(k, I_{\bar{k}}) \leq \delta$ for $k \in (\bar{\theta} - \varepsilon, \bar{k}]$, continuity also implies that $l_{\bar{\theta}}^2 \leq \delta$. From Lemmas 1 and 4, $l_m^1 \geq u_{(m,1]}(k, I_k)$ for $k > \bar{k}$, and $l_m^2 \geq u_{(m,1]}(k, I_k)$ for $k \in (\bar{\theta} - \varepsilon, \bar{k}]$. Then $l_m \equiv \max\{l_m^1, l_m^2\} \geq u_{(m,1]}(k, I_k)$ for $k > \bar{\theta} - \varepsilon$. From Lemma 15, l_m^1 and l_m^2 are continuous in m, and so is l_m. Hence, there exists $m' < \bar{\theta}$ such that $l_{m'} < l_{\bar{\theta}} - \delta/2 \leq \delta/2 < 0$. This implies that

[9] $a(\bar{\theta}) = s(\bar{\theta}, I_{\bar{k}})$, that is, if speculators follow the cutoff rule $I_{\bar{k}}$, the government is indifferent between defending the currency and abandoning the peg at $\theta = \bar{\theta}$.

$u_{(m',1]}(k, I_k) \leq \delta/2$ for $k > \bar{\theta} - \varepsilon$. In this case, either $u_{(m',1]}(k, I_k) < 0$ for all $k \in (m' - \varepsilon, \bar{\theta} - \varepsilon]$, or there exists $k' = \sup\{k \in (m' - \varepsilon, \bar{\theta} - \varepsilon] : u_{(m',1]}(k, I_k) \geq 0\}$. From Lemma 6, either there is no attack on $(m', 1]$, thus $m' \in M$, or, in the worst equilibrium for the government, speculators follow $I_{k'}$ after observing $(m', 1]$. In the latter case, the government abandons the peg for $\theta \leq \theta_{k'} \in (m', \bar{\theta})$. Consider the partition $P^{\theta_{k'}}$. From Lemma 15, $u_{(\theta_{k'},1]}(k, I_k) < 0$ for all $k \in X_{(\theta_{k'},1]}$, and, from Lemma 6, there is no attack on y_h. This means that $\theta_{k'} \in M$. Thus, either $\bar{\theta} > m' \in M$ or $\bar{\theta} > \theta_{k'} \in M$, which implies that $\underline{m} < \bar{\theta}$. $\square$

A.2.10. Proof of Theorem 3

Theorem 3. *Suppose that Assumption 1 is satisfied. For every partition P, $V(P) \leq \overline{V}$, where*

$$\overline{V} = \lim_{m \downarrow \underline{m}} V(P^m) = \sup_{m \in M} V(P^m).$$

Then

i. *if $\underline{m} \in M$, the government's equilibrium payoff is $\overline{V}$. In equilibrium, when $\theta > \underline{m}$, there are no attacks and the peg is maintained; and when $\theta \leq \underline{m}$, every speculator attacks the currency and the peg is abandoned. The government can achieve the payoff $\overline{V}$ with the two-interval partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$.*

ii. *if $\underline{m} \notin M$, no equilibrium exists. However, the government can achieve a payoff arbitrarily close to $\overline{V}$.*

Proof: For any two partitions A and B, if $V(A) > V(B)$, then A is said to be preferred to B. From Lemma 18, we know that $\underline{m} < \bar{\theta}$.

Suppose that the partition P is optimal. From Theorem 1, we can assume that $P = \{0, m, 1\}$.

i. a. Suppose that $m > \underline{m}$. In this case, there exists $m' \in [\underline{m}, m) \cap M$. If $m \leq \theta^*$, from Theorem 2, it follows that the peg is abandoned if and only if $\theta \in [0, m]$. Since

$m' \in M$ and $m' < \theta^*$, the peg is abandoned if and only if $\theta \in [0, m']$. Hence the partition $\{0, m', 1\}$ is preferred to P. If $m > \theta^*$, from Theorem 2, partition $\{0, \theta^*, 1\}$ is preferred to P, a contradiction with the optimality of P. Hence $m \leq \underline{m} < \bar{\theta}$.

b. Suppose that $m < \underline{m}$. Since $m < \bar{\theta}$, the peg is abandoned for $\theta \in [0, m]$. From Lemma 7, in the worst equilibrium for the government, speculators follow a cutoff rule I_{k_h} after observing y_h, where $k_h = \sup\{k \in X_{y_h} : u_{y_h}(k, I_k) \geq 0\}$. Given the speculators' strategy, there exists $\theta_{k_h} > m$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_h}$. From Lemma 15, increasing m would (weakly) decrease the cutoff signal k, which would (weakly) decrease the threshold state θ_k. This implies that, with partition $P' = \{0, \theta_{k_h}, 1\}$, no one attacks if $\theta \in (\theta_{k_h}, 1]$. Thus, P' is preferred to P, a contradiction with the optimality of P. We have that $m = \underline{m}$.

ii. From $a., b.$, if P is an optimal partition, then $m = \underline{m}$. If $\underline{m} \in M$, partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$ is optimal. If $\underline{m} \notin M$, there is no equilibrium, but the government can achieve a payoff arbitrarily close to $\overline{V} = \lim_{m \downarrow \underline{m}} V(P^m)$.

□

A.3. Proofs for Chapter 2

A.3.1. Posteriors

For any pair of continuous random variables A and B, let g_{AB} denote their joint pdf. Let g_A and g_B denote the marginal pdfs, and let $g_{A|B}$ denote the pdf of A conditional on B. Finally, denote the cdfs by G_A and G_B. Following the main text, we denote the pdf of the idiosyncratic noise by g, and its cdf by G, omitting the subscripts.

For $x \in (\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon)$:

$$\begin{aligned} g_{\theta|x}(\theta|x) &= \frac{g_{\theta x}(\theta, x)}{g_x(x)} = \frac{g(x-\theta)g_\theta(\theta)}{\int_{-\infty}^{+\infty} g(x-\tilde{\theta})g_\theta(\tilde{\theta})d\tilde{\theta}} \\ &= \frac{g\left(x-\theta\right)}{G\left(x-\theta_{\min}\right) - G\left(x-\theta_{\max}\right)} \quad \text{, if } \theta \text{ is uniform on } [\theta_{\min}, \theta_{\max}]. \end{aligned}$$

For $x \in \{-\varepsilon, 1+\varepsilon\}$: $\mathbb{P}(\theta = 0|x = -\varepsilon) = 1$; $\mathbb{P}(\theta = 1|x = 1+\varepsilon) = 1$.

A.3.2. Equilibrium Policy

Before proving the results in Section 2.3, I first present some auxiliary results. For $\mu_t \in (0, 1)$:

Claim 1 *Given Assumption 1, $F_H(\delta) < F_L(\delta)$, for $\delta \in (\delta_{min}, \delta_{max})$.*

Proof: Define $\lambda(\delta) \equiv f_H(\delta)/f_L(\delta)$, for all δ. First, notice that $\lambda(\delta_{\min}) < 1$, otherwise

$$\begin{aligned} \lambda(\delta_{\min}) \geq 1, &\quad \text{for } \delta \in (\delta_{\min}, \delta_1) \\ \Rightarrow \lambda(\delta_{\min}) > 1, &\quad \text{for } \delta > \delta_1, \end{aligned}$$

which implies that, for $\delta < \delta_1$

$$F_L(\delta) = \int_{\delta_{\min}}^{\delta} f_L(\tilde{d})d\tilde{\delta} \leq \int_{\delta_{\min}}^{\delta} f_H(\tilde{d})d\tilde{\delta} = F_H(\delta),$$

and for $\delta > \delta_1$

$$\begin{aligned} F_L(\delta) &= F_L(\delta_1) + \int_{\delta_1}^{\delta} f_L(\tilde{d})d\tilde{\delta} \\ &< F_H(\delta_1) + \int_{\delta_1}^{\delta} f_L(\tilde{d})d\tilde{\delta} \\ &= F_H(\delta), \end{aligned}$$

therefore $1 = F_L(\delta_{\max}) < F_H(\delta_{\max}) = 1$, a contradiction.

Define $\underline{\delta} = \inf\{\delta | \lambda(\underline{\delta}) = 1\}$. From Assumption 1, $\underline{\delta}$ is well defined and $\underline{\delta} < \delta_{\max}$, otherwise $\lambda(\delta) < 1$, for all $\delta < \delta_{\max}$, and

$$1 = F_H(\delta_{\max}) = \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta) d\delta < \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta) d\delta = F_L(\delta_{\max}) = 1,$$

a contradiction.

Finally, there exists $\bar{\delta} < \delta_{\max}$, such that $\lambda(\bar{\delta}) > 1$. If this is not the case, then $\lambda(\delta) = 1$ for all $\delta \in (\underline{\delta}, \delta_{\max}]$, therefore

$$\begin{aligned}
1 = F_H(\delta_{\max}) &= F_H(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta) d\delta \\
&= F_H(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta) d\delta \\
&< F_L(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta) d\delta \\
&= F_L(\delta_{\max}) = 1,
\end{aligned}$$

a contradiction.

Thus $f_H(\delta) < f_L(\delta)$, for $\delta \in [\delta_{\min}, \underline{\delta})$; $f_H(\delta) \geq f_L(\delta)$, for $\delta \in (\underline{\delta}, \bar{\delta}]$; and $f_H(\delta) > f_L(\delta)$, for $\delta \in (\bar{\delta}, \delta_{\max}]$. For $\delta \leq \underline{\delta}$, it is clear that $F_H(\delta) < F_L(\delta)$. Suppose that $F_H(\hat{\delta}) = F_L(\hat{\delta})$, for $\hat{\delta} \in (\underline{\delta}, \delta_{\max})$. Then

$$\begin{aligned}
1 = F_L(\delta_{\max}) &= F_L(\hat{\delta}) + \int_{\hat{\delta}}^{\bar{\theta}} f_L(\delta) d\delta + \int_{\bar{\theta}}^{\theta_{\max}} f_L(\delta) d\delta \\
&= F_H(\hat{\delta}) + \int_{\hat{\delta}}^{\bar{\theta}} f_L(\delta) d\delta + \int_{\bar{\theta}}^{\theta_{\max}} f_L(\delta) d\delta \\
&< F_L(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta) d\delta \\
&= F_H(\delta_{\max}) = 1,
\end{aligned}$$

a contradiction. This proves the claim. $\square$

Claim 2 *Given Assumption 1, for $\mu_t \in (0,1)$:*

(i) $\mathbb{E}_\delta[\mu^h_\delta(\mu_t)|H] \geq \mathbb{E}_\delta[\mu^h_\delta(\mu_t)|L]$, *with strict inequality if* $p_I(\mu_t, L) > 0$.

(ii) $\mathbb{E}_\delta[\mu^l_\delta(\mu_t)|H] \leq \mathbb{E}_\delta[\mu^l_\delta(\mu_t)|L]$, *with strict inequality if* $p_I(\mu_t, H) < 1$.

Proof: When $p_I(\mu_t, L) > 0$, and the inefficient government sends signal h with positive probability in state L, the updated reputation following a report $y = h$ and the observation of δ, given by $\mu^h_\delta(\mu_t)$ in (2.7), is strictly increasing in the likelihood ratio $\lambda(\delta) = f_H(\delta)/f_L(\delta)$, and it is constant if $p_I(\mu_t, L) = 0$. Given Assumption 1, if $p_I(\mu_t, L) > 0$

$$
\begin{aligned}
&\int_\Delta \mu^h_\delta(\mu_t) f_H(\delta) d\delta - \int_\Delta \mu^h_\delta(\mu_t) f_L(\delta) d\delta \\
=&\mu^h_\delta(\mu_t)[F_H(\delta) - F_L(\delta)]|^{\delta_{\max}}_{\delta_{\min}} - \int_\Delta \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)] d\delta \\
=&-\int_{\Delta/(\delta_1,\delta_2)} \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)] d\delta - \int_{\delta_1}^{\delta_2} \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)] d\delta, \\
>&0
\end{aligned}
$$

where the inequality comes from the fact that $[F_H(\delta) - F_L(\delta)] < 0$ for $\delta \in (\delta_{\min}, \delta_{\max})$, and because $\lambda(\delta)$ is strictly increasing for $\delta \in (\delta_1, \delta_2)$, and so is $\mu^h_\delta(\mu_t)$. This result implies that the expected value of $\mu^h_\delta(\mu_t)$ is strictly larger in state H than in state L. In other words, the government's expected reputation after a signal h is higher when the the report is truthful and the state is H.

Similarly, if $p_I(\mu_t, H) < 1$, and the government sends signal l with positive probability in state H, the updated reputation $\mu^l_\delta(\mu_t)$ in (2.8) is strictly decreasing in the likelihood ratio $\lambda(\delta)$, and it is constant if $p_I(\mu_t, H) = 1$. Given Assumption 1, $p_I(\mu_t, H) < 1$

$$
\int_\Delta \mu^l_\delta(\mu_t) f_H(\delta) d\delta - \int_\Delta \mu^l_\delta(\mu_t) f_L(\delta) d\delta < 0,
$$

which means that the expected updated reputation after a signal l is higher when the true state is L instead of H. □

Claim 3 *Given Assumption 1, for $\mu_t \in (0,1)$:*

(i) $\bar{\mu}_t(\mu_t, H, h) > \bar{\mu}_t(\mu_t, L, h)$, *with strict inequality if* $p_I(\mu_t, L) > 0$.

(ii) $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, H, l)$, *with strict inequality if* $p_I(\mu_t, H) < 1$.

Proof: Under Assumption 3-A, the realization of δ is always observed and, from (2.10), $\bar{\mu}_t(\mu_t, s, y) = \mathbb{E}_\delta[\mu^y_\delta(\mu_t)|s]$. In this case, therefore the result follows immediately from Claim 2.

Under Assumption 3-B, the realization of δ is only observed if $n \geq N(\theta)$, from (2.12)

$$\bar{\mu}_t(\mu_t, s, y) = P^*(\mu_t, y)\mathbb{E}_\delta[\mu^y_\delta(\mu_t)|s] + [1 - P^*(\mu_t, y)]\mu^y(\mu_t),$$

and the result also follows from Claim 2. □

The intuition behind Claim 3 is the following. Since the efficient government is always truthful, whenever the realization of δ is such that a false report is likely, the entrepreneurs revise their beliefs about the government toward a lower reputation. Hence, if the government send a signal h (l), the expected reputation is lower if true state is L instead of H (H instead of L).

Proof of Lemma 9

Let $\mu_t \in (0,1)$. It is sufficient to show that $G_H > 0$, where G_H is the gain from truthful disclosure in state H, given by (2.13). Suppose that $G_H \leq 0$. Then,

$$\bar{\mu}_t(\mu_t, L, l) \geq \bar{\mu}_t(\mu_t, H, l) \geq \bar{\mu}_t(\mu_t, H, h) \geq \bar{\mu}_t(\mu_t, L, h), \tag{A.5}$$

where the first and last inequalities come from Claim 3, and the second one follows from $G_H \leq 0$. If $p_I(\mu_t, L) > 0$, from Claim 3 the last inequality in (A.5) is strict, therefore $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, L, h)$. This implies that G_L, given by (2.14), is strictly positive, and therefore the government only sends signal l in state L, a contradiction with $p_I(\mu_t, L) > 0$. Hence $p_I(\mu_t, L) = 0$, and from (2.7)

$$\mu_\delta^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I p_I(\mu_t, H)(1-\mu)} = \mu^h(\mu_t),$$

where $\mu^h(\mu_t)$ is given by (2.3). From (2.10) and (2.12), it follows that $\bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t)$.

To get a contradiction, I need to show that $\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h)$, which implies that $G_H > 0$. Notice that $\mu^h(\mu_t)$ is strictly decreasing in $p_I(\mu_t, H)$, and from (2.4) and (2.8), both $\mu^l(\mu_t)$ and $\mu_\delta^l(\mu_t)$ are strictly increasing in $p_I(\mu_t, H)$, and so is $\bar{\mu}(\mu_t, H, l)$. It suffices to show that, for $p_I(\mu_t, H) = 1$, $\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t)$.

If $p_I(\mu_t, H) = 1$

$$\bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I (1-\mu)},$$

and from (2.4), (2.8), (2.10) and (2.12)

$$\bar{\mu}(\mu_t, H, l) = \mu^l(\mu_t) = \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)}.$$

Then

$$\begin{aligned}
&\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h) \\
\Leftrightarrow \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I (1-\mu)} > \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)} & \\
&\Leftrightarrow \pi_E[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)] > (1-\pi_E)[\pi_E \mu_t + \pi_I(1-\mu_t)] \\
&\Leftrightarrow \pi_E(1-\pi_I) > (1-\pi_E)\pi_I \\
&\Leftrightarrow \frac{(1-\pi_I)}{(1-\pi_E)} > \pi_I \pi_I,
\end{aligned}$$

which is true, since $\pi_E > \pi_I$. Thus $G_H > 0$, a contradiction.

Proof of Lemma 10

Let $p_\mu \equiv p_I(\mu, L)$. From Lemma 9, $p_I(\mu_t, H, h) = 1$.

(i). From (2.3),(2.4),(2.7),(2.8), (2.10) and (2.12), $\bar{\mu}(0, s, y) = 0$, for all s and y, and $\bar{\mu}(1, s, y) = 1$, for all s and y. Thus $G(0, p) = G(1, p) = 0$.

(ii).

$$G_L(\mu, 0) = \frac{(1-\pi_E)\mu}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)} - \frac{\pi_E \mu}{\pi_E \mu_t + \pi_I(1-\mu)}, \qquad \text{(A.6)}$$

then

$$G_L(\mu, 0) < 0 \Leftrightarrow \frac{(1-\pi_I)}{(1-\pi_E)} > \pi_I \pi_I,$$

which holds, since $\pi_E > \pi_I$.

(iii). If $p_\mu = 1$, then the inefficient government always sends $y = h$. In this case, entrepreneurs are sure that the government is efficient when $y = l$, but are uncertain about the type when $y = h$. Thus $\bar{\mu}(\mu, s, L) = 1$ and $\bar{\mu}(\mu, L, h) < 1$, which implies that $G_L(\mu, 1) > 0$.

(iv). From (A.6)

$$\frac{\partial}{\partial \mu} G_L(\mu, 0) = \frac{(1-\pi_E)(1-\pi_I)}{[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)]^2} - \frac{\pi_E \pi_I}{[\pi_E \mu_t + \pi_I(1-\mu)]^2},$$

and

$$\frac{\partial^2}{\partial \mu^2} G_L(\mu, 0) = 2\frac{(1-\pi_E)(1-\pi_I)(\pi_E - \pi_I)}{[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)]^3} + 2\frac{\pi_E \pi_I (\pi_E - \pi_I)}{[\pi_E \mu_t + \pi_I(1-\mu)]^3} > 0.$$

Proof of Lemma 11

Let $\mu_t \in (0, 1)$. From Lemma 10 part (ii), if entrepreneurs believe that $p_\mu = 0$, then the government is strictly better off by deviating and sending signal $y = h$. From Lemma 10 part (iii), if entrepreneurs believe that $p_\mu = 1$, then the government is strictly better off by deviating and sending signal $y = l$ in state L. If an equilibrium exists, then $p_\mu \in (0,1)$, and the government must be indifferent between sending signals h and l when the state is L, which implies that $G_L(\mu, p_\mu) = 0$. From Lemma 10 parts (ii) and (iii), and from the continuity of $G_L(\mu, p)$ in p, there exists $p_\mu \in (0,1)$ such that $G_L(\mu, p_\mu) = 0$, therefore an equilibrium exists.

Under Assumption 3-A,

$$\begin{aligned} G_L(\mu, p_\mu) =& \frac{(1-\pi_E)\mu}{(1-\pi_E)\mu_t + (1-\pi_I)(1-p_\mu)(1-\mu)} \\ & - \mathbb{E}_\delta \left[\left. \frac{\pi_E \mu_t}{\pi_E \mu + \left[\pi_I + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_I)p_\mu\right](1-\mu)} \right| L \right], \end{aligned}$$

thus $G_L(\mu, p_\mu)$ is strictly increasing in p_μ. In this case, there exists a unique $p^*_\mu \in (0,1)$ that solves $G_L(\mu, p_\mu) = 0$.

Proof of Proposition 3

Let $\mu_t \in (0,1)$. From Lemma 11, if an equilibrium where the efficient government follows a full disclosure policy exists, the inefficient government's strategy for period t in such an equilibrium is given by $p_I(\mu_t, H) = 1$ and $p_I(\mu, L) = p_\mu \in (0,1)$, where p_μ solves $G_L(\mu, p_\mu) = 0$. It is left to show that given the inefficient government's strategy and the entrepreneurs' beliefs, it is indeed optimal for the efficient government to be truthful. If entrepreneurs believe that the efficient government is truthful, then:(1) in the proof of Lemma 9 I show that $G_H > 0$; (2) and from Lemma 11, the inefficient government chooses p_μ such that $G_L(\mu, p_\mu) = 0$. From $G_H > 0$, the efficient government strictly prefers to be truthful in state H, and from $G_L = 0$, the efficient government is indifferent in state L. Thus an equilibrium where the efficient government is always truthful exists. Furthermore, if Assumption 3-A holds, from Lemma 11, the equilibrium is unique, since there exists a unique p^*_μ that solves $G_L(\mu, p_\mu) = 0$.

Proof that there is no equilibrium where type I follows a full disclosure policy

Lemma 19 *Let $\mu_t \in (0,1)$. In equilibrium, the inefficient government never follows a full disclosure policy in period t. There is no equilibrium where*

$$p_I(\mu_t, H) = 1 - p_I(\mu_t, L) = 1.$$

Proof: If the inefficient government is always truthful, then

$$\mu_\delta^h(\mu_t) = \frac{\left[\pi_E p_E(\mu_t, H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_E) p_E(\mu_t, L)\right]\mu_t}{\left[\pi_E p_E(\mu_t, H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_E) p_E(\mu_t, L)\right]\mu_t + \pi_I(1-\mu)},$$

and

$$\mu_\delta^l(\mu_t) = \frac{\left[\frac{f_H(\delta)}{f_L(\delta)}\pi_E(1-p_E(\mu_t, H)) + (1-\pi_E)(1-p_E(\mu_t, L))\right]\mu_t}{\left[\frac{f_H(\delta)}{f_L(\delta)}\pi_E(1-p_E(\mu_t, H)) + (1-\pi_E)(1-p_E(\mu_t, L))\right]\mu_t + (1-\pi_I)(1-\mu_t)},$$

therefore $\mu_\delta^h(\mu_t)$ is strictly decreasing in $\lambda(\delta) = f_H(\delta)/f_L(\delta)$ if $p_E(\mu_t, L) > 0$, and constant otherwise; $\mu_\delta^l(\mu_t)$ is strictly increasing in $\lambda(\delta)$ if $p_E(\mu_t, H) < 0$, and constant otherwise. For $\mu_t \in (0,1)$, following similar arguments to those in Claim 3, Assumption 1 implies:

(A) $\bar{\mu}_t(\mu_t, H, h) < \bar{\mu}_t(\mu_t, L, h)$, with strict inequality if $p_E(\mu_t, L) > 0$.

(B) $\bar{\mu}_t(\mu_t, L, l) < \bar{\mu}_t(\mu_t, H, l)$, with strict inequality if $p_E(\mu_t, H) < 1$.

This means that if the efficient government is the only type that might not be truthful, the government's reputation increases whenever the realization of δ is such that a false report is likely. If the government send a signal h (l), the expected reputation is higher if the true state is L instead of H (H instead of L).

If the inefficient government is truthful, then $G_H \geq 0$, which implies that

$$\bar{\mu}_t(\mu_t, L, h) \geq \bar{\mu}_t(\mu_t, H, h) \geq \bar{\mu}_t(\mu_t, H, l) \geq \bar{\mu}_t(\mu_t, L, l), \tag{A.7}$$

where the first and last inequalities come from (A) and (B) above, and the second one follows from $G_H \geq 0$. If either $p_E(\mu_t, L) > 0$ or $p_E(\mu_t, H) < 1$, from (A) and (B), either the first or the third inequalities in (A.7) are strict, therefore and $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, L, h)$. This implies that both $G_H > 0$ and $G_L > 0$, thus the inefficient government is always truthful. However, from Lemma 11, there is no equilibrium in which both types of government are always truthful, thus there is no equilibrium where the inefficient government is truthful. $\square$

A.3.3. Equilibrium of the game between entrepreneurs

In this section, I characterize the equilibrium of the game between entrepreneurs, conditional on an expected value of δ given by $\bar{\delta}$. I provide results that will be used to prove Proposition 4. The results in this section are based on Galvao and Shalders (2017).

Lemma 20 *For a given public signal y, if $\pi(x, y) \geq \pi'(x, y)$ for all x, then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x.*

Proof:

$$\begin{aligned} \eta(\bar{\delta}, x) \geq \eta'(\bar{\delta}, x) \forall x \Rightarrow n(\bar{\delta}, \theta, \eta) \geq n(\bar{\delta}, \theta, \eta') \forall \theta \Rightarrow A(\bar{\delta}, \eta) \supseteq A(\bar{\delta}, \eta') \\ \Rightarrow u(\bar{\delta}, x, \eta) \geq u(\bar{\delta}, x, \eta'). \end{aligned}$$

$\square$

For $k \in [\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon]$, let the indicator function I_k be defined as

$$I_k(x) = \begin{cases} 1, & \text{if } x \leq k \\ 0, & \text{if } x > k \end{cases} . \tag{A.8}$$

Suppose that the investment strategies are given by $a_i(\bar{\delta}, x_i) = I_k(x_i)$, for all i: entrepreneurs follow a cutoff strategy, investing if and only if $x_i \leq k$. The number of ventures is thus given by

$$n(\bar{\delta}, \theta, I_k) = G(k - \theta). \tag{A.9}$$

Note that $n(\bar{\delta}, \theta, I)$ is strictly decreasing in θ for $\theta \in (k - \varepsilon, k + \varepsilon)$, and constant otherwise. Let

$$t_k \equiv \sup\{\theta | n(\bar{\delta}, \theta, I) \geq N(\theta)\},$$

and let $\theta_k = \min\{t_k, \theta_{\min}\}$. If the probability of failure is below t_k when entrepreneurs follow I_k, then the number of ventures is large enough so that the successful ventures pay $v + \delta$. If $k \in (\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon)$, there is a unique θ such that $n(\bar{\delta}, \theta, I) = G(k - \theta) = N(\theta)$, and therefore $\theta_k = k - G^{-1}(N(\theta_k))$.

Let $\psi(k) = \theta_k - k$. The following lemma characterizes θ_k and $\psi(k)$.

Lemma 21 *(i) The function $\psi(\cdot)$ is continuous and decreasing, with $\psi(k) \in [-\varepsilon, \varepsilon]$, for all k.*

(ii) For $k \in (\theta_{min} + \varepsilon, \theta_{max} - \varepsilon)$, $\psi(\cdot)$ is differentiable, with derivative $\psi'(k) > -1$.

(iii) θ_k is increasing in k, for all k.

Proof: Let $\underline{k}$ solve $G(\underline{k} - \theta) = N(\theta_{\min})$. Then $\underline{k} = G^{-1}(N(\theta_{\min})) + \theta_{\min} \in (\theta_{\min} - \varepsilon, \theta_{\min} + \varepsilon)$. If $k < \underline{k}$, then for all θ

$$N(\theta) \geq N(\theta_{\min}) = G(\underline{k} - \theta) \geq G(k - \theta) \Rightarrow \theta_k = \theta_{\min} \in (k - \varepsilon, k + \varepsilon).$$

Let $\bar{k}$ solve $G(\bar{k} - \theta_{\max}) = N(\theta_{\max})$. Then $\bar{k} = G^{-1}(N(\theta_{\max})) + \theta_{\max} \in (\theta_{\max} - \varepsilon, \theta_{\max} + \varepsilon)$. If $k > \bar{k}$, then for all θ

$$G(k - \theta_{\max}) \geq G(\bar{k} - \theta_{\max}) = N(\theta_{\max}) \Longrightarrow \theta_k = \theta_{\max} \in (k - \varepsilon, k + \varepsilon).$$

For $k \in (\underline{k}, \bar{k})$, we have $\theta_k = k - G^{-1}(N(\theta_k)) \in (k + \varepsilon, k + \varepsilon)$. The function $\psi(k) = \theta_k - k$ is then given by

$$\psi(k) = \begin{cases} \theta_{\max} - k, & \text{if } k < \underline{k} = \theta_{\min} + G^{-1}(N(\theta_{\min})) \\ -G^{-1}(N(\theta_k)), & \text{if } \underline{k} \leq k \leq \bar{k} \\ \theta_{\min} - k, & \text{if } k > \bar{k} = \theta_{\max} + G^{-1}(N(\theta_{\max})) \end{cases}. \tag{A.10}$$

From (A.10), it is clear that $\psi(k)$ is continuous in k. Since $N(\theta)$ is increasing in θ, then θ_k is increasing in k, which implies that $\psi(k)$ is decreasing in k. Since $k \in (\theta_k - \varepsilon, \theta_k + \varepsilon)$, then $\psi(k) \in (-e, +\varepsilon)$, and part (i) is proved. If $k \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon) \subseteq (\underline{k}, \bar{k})$,

$$\begin{aligned} \psi(k) = -G^{-1}(N(\theta_k)) \Rightarrow \psi'(k) &= -\frac{N'(k + \theta_k)}{g(G^{-1}(N(k + \theta_k)))}(\psi'(k) + 1) \\ &= -\frac{N'(k + \theta_k)}{N'(k + \theta_k) + g(G^{-1}(N(k + \theta_k)))} \in (-1, 0], \end{aligned}$$

which proves part (ii). Finally, for $k \in (\underline{k}, \bar{k})$, θ_k is differentiable, with derivative $1 - \psi'(k) > 0$, and it is constant otherwise. This proves part (iii). $\square$

From 1.12 and the definition of ψ, the expected payoff for the entrepreneur who observed the cutoff signal k is given by

$$u(\bar{\delta}, k, I_k) = v \int_{k-\varepsilon}^{k+\varepsilon} (1 - \theta)\phi(\theta|k)d\theta + \bar{\delta} \int_{k-\varepsilon}^{k+\psi(k)} (1 - \theta)\phi(\theta|k)d\theta. \tag{A.11}$$

Since $\phi(\cdot|k)$ and the limits of integration in (A.11) are continuous in k (because $\psi(\cdot)$ is continuous), $u(\bar{\delta}, k, I_k)$ is continuous in the cutoff k.

Lemma 22 *For $k \in (\theta_{min}+\varepsilon, \theta_{max}-\varepsilon)$, the payoff function $u(\bar{\delta}, k, I_k)$ is strictly decreasing in k.*

Proof: From (1.2) and (A.11), the payoff function is given by

$$\begin{aligned} u(\bar{\delta}, k, I_k) =& v \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) \frac{g(k-\theta)}{G(k-\theta_{\min}) - G(k-\theta_{\max})} d\theta \\ &+ \bar{\delta} \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) \frac{g(k-\theta)}{G(k-\theta_{\min}) - G(k-\theta_{\max})} d\theta. \end{aligned} \tag{A.12}$$

From (A.10), $\psi(\cdot)$ is differentiable in k for $k \in (\theta_{\min}+\varepsilon, \theta_{\max}-\varepsilon)$, and so is $u(\bar{\delta}, k, I_k)$. Differentiating $u(\bar{\delta}, k, I_k)$ with respect to k and using the fact that $G(k-\theta_{\max}) = g(k-\theta_{\max}) = 0$, for $k < \theta_{\max} - \varepsilon$, yield

$$\begin{aligned} &\frac{d}{dk} u(\bar{\delta}, k, I_k) \\ =& \frac{v}{G(k-\theta_{\min})} \left[(1-k-\varepsilon) g(-\varepsilon) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) g'(k-\theta) d\theta \right. \\ & \left. \qquad - \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) \frac{g(k-\theta) g(k-\theta_{\min})}{G(k-\theta_{\min})} d\theta \right] \\ &+ \frac{\bar{\delta}}{G(k-\theta_{\min})} \left[(1-k-\psi(k)) g(-\psi(k))(1+\psi'(k)) - (1-k+\varepsilon) g(\varepsilon) \right. \\ & \left. \qquad + \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) g'(k-\theta) d\theta - \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) \frac{g(k-\theta) g(k-\theta_{\min})}{G(k-\theta_{\min})} d\theta \right] \\ \le& \frac{v}{G(k-\theta_{\min})} \left[(1-k-\varepsilon) g(-\varepsilon) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) g'(k-\theta) d\theta \right] \\ &+ \frac{\bar{\delta}}{G(k-\theta_{\min})} \left[(1-k-\psi(k)) g(-\psi(k)) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) g'(k-\theta) d\theta \right], \end{aligned}$$

where the inequality comes from $\psi'(k) \le 0$, and from the fact that the second and fourth integrals on the RHS of the equality are positive. Define $\underline{\tilde{\varepsilon}}$ as

$$\underline{\tilde{\varepsilon}} = \inf\{\tilde{\varepsilon} \in [-\varepsilon, \varepsilon] : g'(\hat{\varepsilon}) \le 0 \quad \forall \hat{\varepsilon} > \tilde{\varepsilon}\}.$$

From (1.1), $\underline{\tilde{\varepsilon}}$ is well defined. Furthermore, $g'(\tilde{\varepsilon}) \ge 0$, for $\tilde{\varepsilon} \le \underline{\tilde{\varepsilon}}$, and $g'(\tilde{\varepsilon}) \le 0$, for $\tilde{\varepsilon} > \underline{\tilde{\varepsilon}}$.

Define $\underline{\tilde{\theta}}$ as

$$\underline{\tilde{\theta}} = k - \underline{\tilde{\varepsilon}}.$$

Hence $\underline{\tilde{\theta}} \in [k-\varepsilon, k+\varepsilon]$. We then have

$$\begin{aligned}\int_{k-\varepsilon}^{k+\varepsilon}(1-\theta)g'(k-\theta)d\theta &\leq (1-\underline{\tilde{\theta}})\int_{k-\varepsilon}^{\underline{\tilde{\theta}}} g'(k-\theta)d\theta + (1-\underline{\tilde{\theta}})\int_{\underline{\tilde{\theta}}}^{k+\varepsilon} g'(k-\theta)d\theta \\ &= (1-\underline{\tilde{\theta}})[g(\varepsilon)-g(-\varepsilon)],\end{aligned}$$

and

$$\begin{aligned}\int_{k-\varepsilon}^{k+\psi(k)}(1-\theta)g'(k-\theta)d\theta \leq &(1-\min\{\underline{\tilde{\theta}}, k+\psi(k)\})\int_{k-\varepsilon}^{\min\{\underline{\tilde{\theta}},k+\psi(k)\}} g'(k-\theta)d\theta \\ &+ (1-\min\{\underline{\tilde{\theta}}, k+\psi(k)\})\int_{\min\{\underline{\tilde{\theta}},k+\psi(k)\}}^{k+\psi(k)} g'(k-\theta)d\theta \\ =&(1-\min\{\underline{\tilde{\theta}}, k+\psi(k)\})[g(\varepsilon)-g(-\psi(k))].\end{aligned}$$

Hence

$$\begin{aligned}&\frac{d}{dk}u(\bar{\delta}, k, I_k) \\ \leq&\frac{v}{G(k-\theta_{\min})}\Big[(1-k-\varepsilon)g(-\varepsilon)-(1-k+\varepsilon)g(\varepsilon)+(1-\underline{\tilde{\theta}})[g(\varepsilon)-g(-\varepsilon)]\Big] \\ &+\frac{\bar{\delta}}{G(k-\theta_{\min})}\big[(1-k-\psi(k))g(-\psi(k))-(1-k+\varepsilon)g(\varepsilon) \\ &\qquad +(1-\min\{\underline{\tilde{\theta}}, k+\psi(k)\})[g(\varepsilon)-g(-\psi(k))]t\Big] \\ =&\frac{v}{G(k-\theta_{\min})}\Big[g(-\varepsilon)[\underline{\tilde{\theta}}-(k+\varepsilon)]-g(\varepsilon)[\underline{\tilde{\theta}}-(k-\varepsilon)]\Big] \\ &+\frac{\bar{\delta}}{G(k-\theta_{\min})}\Big[g(-\psi(k))[\min\{\underline{\tilde{\theta}}, k+\psi(k)\}-(k+\psi(k))] \\ &\qquad -g(\varepsilon)[\min\{\underline{\tilde{\theta}}, k+\psi(k)\}-(k-\psi(k))]\Big] \\ <&\, 0,\end{aligned}$$

which implies that $u(\bar{\delta}, k, I_k)$ is strictly decreasing. $\square$

Proof of Proposition 4

Using Lemma 22, the proof of existence and uniqueness of equilibrium in the game between entrepreneurs is analogous to the one in Morris and Shin (1998), Theorem 1. Entrepreneurs follow a cutoff rule in their private signal given by $I_{x^*(\bar{\delta})}$, where $x^*(\bar{\delta})$ is such that

$$u(\bar{\delta}, x^*(\bar{\delta}), I_{x^*(\bar{\delta})}) = w, \tag{A.13}$$

which means that the entrepreneur that receives the cutoff signal is indifferent between investing and working. Since $2\varepsilon < \min\{\underline{\theta} - \theta_{\min}, \theta_{\max} - \bar{\theta}_{\delta_H}\}$, then $x^*(\bar{\delta}) \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon)$. The equilibrium number of ventures is

$$n(\bar{\delta}, \theta, I_{x^*(\bar{\delta})}) = G(x^*(\bar{\delta}) - \theta),$$

which is decreasing in θ. The threshold θ below which $n(\bar{\delta}, \theta, I_{x^*(\bar{\delta})}) \geq N(\theta)$ is given by $\theta^*(\bar{\delta}) \equiv \theta_{x^*(\bar{\delta})}$. From (A.11), it is clear that $u(\bar{\delta}, k, I_k)$ is strictly increasing in $\bar{\delta}$, for all k. Lemma 22 and (A.13) thus imply that $x^*(\bar{\delta})$ is strictly increasing in δ. Finally, from Lemma 21, part (iii), $\theta^*(\bar{\delta})$ is also strictly increasing in $\bar{\delta}$.

A.3.4. Welfare function

This section presents properties of the welfare function and establishes results used to prove Lemmas 12, 13, and 14.

Since $x^*(\bar{\delta}) \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon)$, then $G(x^*(\bar{\delta}) - \theta_{\min}) \geq G(\varepsilon) = 1$, and $G(x^*(\bar{\delta}) - \theta_{\max}) \leq G(-\varepsilon) = 0$. From (A.12), the expected payoff after observing $x^*(\bar{\delta})$ can be written as

$$u(\bar{\delta}, x^*(\bar{\delta}), I_{x^*(\bar{\delta})}) = v \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta + \bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta,$$

and the indifference condition (A.13) implies

$$\begin{aligned}
&v \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1-\theta) g(x^*(\bar{\delta})-\theta) d\theta + \bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta})-\theta) d\theta = w \\
&\Rightarrow \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} [(1-\theta)v - w] g(x^*(\bar{\delta})-\theta) d\theta = -\bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta})-\theta) d\theta.
\end{aligned} \tag{A.14}$$

For $s \in \{H, L\}$, define the function $V_s(x^*)$ as

$$\begin{aligned}
V_s(x^*) =& (v+\delta_s) \int_{\theta_{\min}}^{x^*-\varepsilon} (1-\theta) d\theta + \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)(v+\delta_s) - w] G(x^*-\theta) d\theta \\
&+ \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)v - w] G(x^*-\theta) d\theta + w \int_{x^*-\varepsilon}^{\theta_{\max}} d\theta.
\end{aligned}$$

Thus $V_s(x^*(\bar{\delta})) = W_s(\bar{\delta})$, for all $\bar{\delta}$, where $W_s(\bar{\delta})$ is the welfare function given by (2.22).

From Lemma 21, part (ii), $\psi(k)$ is differentiable at $x^*(\bar{\delta})$, and $\psi'(x^*(\bar{\delta})) > -1$. Hence $V_s(x^*)$ differentiable:

$$\begin{aligned}
&\frac{\partial}{\partial x^*} V_s(x^*) = \\
&(v+\delta_s) \left\{ 1 - x^* + \varepsilon + [1 - x^* - \psi(x^*)][1+\psi'(x^*)] G(-\psi(x^*)) - (1 - x + \varepsilon) G(\varepsilon) \right\} \\
&+ v \left\{ (1 - x^* - \varepsilon) G(-\varepsilon) - [1 - x^* - \psi(x^*)][1+\psi'(x^*)] G(-\psi(x^*)) \right\} \\
&- w \left\{ G(-\psi(x^*))[1+\psi'(x^*)] - G(\varepsilon) - G(-\varepsilon) - G(-\psi(x^*))[1+\psi'(x^*)] + 1 \right\} \\
&+ \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)(v+\delta_s) - w] g(x^*-\theta) d\theta + \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)v - w] g(x^*-\theta) d\theta \\
=& \int_{x^*-\varepsilon}^{x^*+\varepsilon} [(1-\theta)v - w] g(x^*-\theta) d\theta \\
&+ \delta_s \left\{ [1 - x^* - \psi(x^*)][1+\psi'(x^*)] G(-\psi(x^*)) + \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} (1-\theta) g(x^*-\theta) d\theta \right\}.
\end{aligned} \tag{A.15}$$

Using (A.14),

$$\left.\frac{\partial}{\partial x^*}V_s(x^*)\right|_{x^*=x^*(\bar{\delta})} \tag{A.16}$$

$$= (\delta_s - \bar{\delta})\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta + \delta_s[1-x^*-\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*)). \tag{A.17}$$

Proof of Lemma 12

From (A.16)

$$\left.\frac{\partial}{\partial x^*}V_H(x^*)\right|_{x^*=x^*(\bar{\delta})}$$

$$= (\delta_H - \bar{\delta})\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta + \delta_H[1-x^*+\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*)).$$

Since $\delta_H \geq \bar{\delta}$ (with strict inequality when entrepreneurs assign a positive probability to L), and $\psi'(x^*(\bar{\delta})) > -1$, then

$$\left.\frac{\partial}{\partial x^*}V_H(x^*)\right|_{x^*=x^*(\bar{\delta})} \geq 0,$$

with strict inequality if $\bar{\delta} < \delta_H$. From $V_H(x^*(\bar{\delta})) = W_H(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\frac{\partial}{\partial\bar{\delta}}W_H(\bar{\delta}) = \left.\frac{\partial}{\partial x^*}V_H(x^*)\right|_{x^*=x^*(\bar{\delta})}\frac{\partial x^*(\bar{\delta})}{\partial\bar{\delta}} \geq 0,$$

with with strict inequality if $\bar{\delta} < \delta_H$.

Proof of Lemma 13

From (A.16)

$$\frac{\partial}{\partial x^*} V_L(x^*)\bigg|_{x^*=x^*(\bar{\delta})}$$
$$= (\delta_L - \bar{\delta}) \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} (1-\theta) g(x^*-\theta) d\theta + \delta_L [1 - x^* - \psi(x^*)][1+\psi'(x^*)] G(-\psi(x^*)).$$

Since $[1-(x^*+\psi(x^*))] > (1-\delta)$, for $\theta > x^* + \psi(x^*)$, and

$$G(-\psi(x^*)) = \int_{x^*+\psi(x^*)}^{x^*+\varepsilon} g(x^*-\theta) d\theta, \tag{A.18}$$

then

$$\frac{\partial}{\partial x^*} V_L(x^*)\bigg|_{x^*=x^*(\delta_L)} \geq \delta_L [1+\psi'(x^*(\delta_L))] \int_{x^*(\delta_L)+\psi(x^*(\delta_L))}^{x^*(\delta_L)+\varepsilon} (1-\theta) g(x^*(\delta_L)-\theta) d\theta > 0.$$

From $V_L(x^*(\bar{\delta})) = W_L(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\frac{\partial}{\partial \bar{\delta}} W_L(\bar{\delta})\bigg|_{\bar{\delta}=\delta_L} = \frac{\partial}{\partial x^*} V_L(x^*)\bigg|_{x^*=x^*(\delta_L)} \frac{\partial x^*(\bar{\delta})}{\partial \bar{\delta}}\bigg|_{\bar{\delta}=\delta_L} > 0.$$

Proof of Lemma 14

From (A.15) and (A.18)

$$
\begin{aligned}
&\frac{\partial}{\partial x^*} V_L(x^*)\bigg|_{x^*=x^*(\delta_H)} \\
=&\int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)(v+\delta_l)-w]g(x^*-\theta)d\theta + \int_{x^*+\psi(x^*)}^{x^*+\varepsilon} [(1-\theta)v-w]g(x^*-\theta)d\theta \\
&+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
<&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon} [(1-\theta)v-w]g(x^*-\theta)d\theta \\
&+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
\leq&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon} [(1-(x^*+\psi(x^*)))v-w]g(x^*-\theta)d\theta \\
&+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
=&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon} [(1-(x^*+\psi(x^*)))(v+\delta_L)-w]g(x^*-\theta)d\theta \\
&+\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
<&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon} [(1-\bar{\theta}_L)(v+\delta_L)-w]g(x^*-\theta)d\theta \\
&+\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
=&\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \\
&<0.
\end{aligned}
$$

The first inequality follows from $(1-\theta)(v+\delta_L) < w$, for $\theta > \bar{\theta}_L$ and $x^*(\delta_H) - \varepsilon > \bar{\theta}_L$. The third inequality is obtained from $x^*(\delta_H) + \psi(x^*(\delta_H)) > x^*(\delta_H) - \varepsilon > \bar{\theta}_L$. The fourth inequality follows from the definition of $\underline{\theta}_L$: $(1-\underline{\theta}_L)(v+\delta_L) = w$. Finally, the last inequality follows from $\psi'(x^*(\delta_H)) < 0$. From $V_L(x^*(\bar{\delta})) = W_L(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\left.\frac{\partial}{\partial\bar{\delta}}W_L(\bar{\delta})\right|_{\bar{\delta}=\delta_H} = \left.\frac{\partial}{\partial x^*}V_L(x^*)\right|_{x^*=x^*(\delta_H)} \left.\frac{\partial x^*(\bar{\delta})}{\partial\bar{\delta}}\right|_{\bar{\delta}=\delta_H} < 0.$$

From the continuity of W_L, there exists $\tilde{\delta}$ such that $\frac{\partial}{\partial\bar{\delta}}W_L(\bar{\delta}) < 0$, for $\bar{\delta} > \tilde{\delta}$.

A.4. Credit Market

This section drops the assumption that only labor is necessary to start a new venture. Now a venture also requires one unit of capital, which is borrowed in a perfectly competitive credit market. There exists an equilibrium for the model with capital where the investment decisions are the same as the equilibrium decisions in the model without capital, as described in Proposition 4. In this equilibrium, the welfare results from Section 2.4 still hold.

There are two types of agents: entrepreneurs and lenders. The agents' problem in each period is now similar to the one in Veldkamp (2005).[10] In each period, the entrepreneurs now have to borrow one unit of capital to invest in a new venture. An entrepreneur that does not invest works for a fixed wage $\tilde{w}$. There is a continuum of lenders, who are index by j and uniformly distributed on $[0, J]$, with $J > 1$.[11] As the entrepreneurs, lenders are infinitely-lived, risk-neutral profit maximizers. At the beginning of each period, lenders can either use one indivisible unit of capital to buy a risk-free bond which pays a return of $(1 + r)$ at the end of the period, or they can lend capital to an entrepreneurs. The risk-free rate is exogenous and constant. The lender receives $(1 + \rho)$ at the end of the period if the venture is successful, and nothing otherwise. The market lending rate is endogenous and depends on the expected rate of default. It is assumed that, when entrepreneur i and a

[10] In her paper, there is a finite number of entrepreneurs and lenders, who are infinitely lived, risk-neutral, and profit maximizers. There are more lenders than entrepreneurs, and the credit market is perfectly competitive. In each period, entrepreneurs can either borrow one unit of capital to invest in a new venture, or work for a fixed wage. Successful ventures pay v_i to entrepreneur i. The probability of success in each period is the same for all new ventures, and it depends on an unobservable and persistent state variable. Lenders can either invest one unit of capital in a risk-free bond that pays $(1 + r)$, or lend it to potential borrowers, who pay $(1 + \rho)$ in case of success, and nothing otherwise. In equilibrium, since lenders are perfectly competitive, the expected return from lending is the risk-free rate: $\mathbb{P}(success)(1 + \rho) = 1 + r$.

[11] There are more lenders than entrepreneurs.

lender j meet, the lender can perfectly observe the entrepreneur's private signal about the probability of failure, x_i.

A Markov strategy for lender j is $\rho_j : [0,1] \times Y \times X \to \mathbb{R}$, where $\rho_j(\mu_t, y, x)$ is the interest rate that lender j charges from an entrepreneur who received a signal x, conditional on (μ_t, y). Given a reputation μ_t and a public signal y, agents form beliefs about the state and lenders announce a pricing function $\rho_j(\mu_t, y, x)$. Entrepreneurs can choose which lenders to borrow from, but lenders cannot commit to an interest rate. Once lender j observes x_i, he can decide not to lend to entrepreneur i. In this case, the lender buys the risk-free bond, while the entrepreneur can search for another lender. Interest rate $\rho_j(\mu_t, y, x_i)$ is only credible if lender j's expected payoff conditional on (μ_t, y, x_i) is greater than $(1 + r)$.

Apart from the introduction of the lenders and the requirement that one unit of capital must be borrowed to start a new venture, the model is the same as in Section 1.3. The timing in period t is as follows:

1. Reputation starts at μ_t.

2. Nature draws $s \in \{H, L\}$.

3. The government observes s and sends a signal $y \in \{h, l\}$.

4. Agents form beliefs about the state and lenders announce pricing functions

$$\{\rho_j(\mu_t, y, \cdot)\}_{j=\in[0,J]}.$$

5. Nature draws the probability of failure θ.

6. Entrepreneurs observe interest rates and private signals about θ, and decide whether or not borrow.

7. If entrepreneur i and lender j agree on a loan, i borrows at rate $\rho_j(\mu_t, y, x_i)$.

8. Lenders not matched with borrowers invest in the risk-free bond. Entrepreneurs that do not invest receive a wage $\tilde{w}$.

9. The outcomes of all ventures are publicly observed, payoffs are received, and the reputation is updated to μ_{t+1}.

Let $\bar{\delta}(\mu_t, y) = \bar{\delta}$, and let the measure of entrepreneurs who invest, given $\bar{\delta}$ and a private signal x, be denoted by $\eta(\mu_t, y, x)$. The number of ventures is characterized in (2.15), and the event where ventures pay $(v + \delta)$ is given by $A(\mu_t, y, \eta)$, described in (2.17). Lender $j's$ expected payoff from lending to an entrepreneur who receives private signal x is thus

$$\begin{aligned} R_j(\mu_t, y, x, \eta) = & \min\{1 + \rho_j(\mu_t, y, x), v\} \int_{[x-\varepsilon, x+\varepsilon]/A(\mu_t, y, \eta)} (1 - \theta)\phi(\theta|x)d\theta \\ & + \mathbb{E}_\delta[\min\{1 + \rho_j(\mu_t, y, x), v + \delta\}|\mu_t, y] \int_{[x-\varepsilon, x+\varepsilon]\cap A(\mu_t, y, \eta)} (1 - \theta)\phi(\theta|x)d\theta. \end{aligned} \tag{A.19}$$

In equilibrium, lender j enters into a contract with an entrepreneurs who receives a signal x if

$$R_j(\mu_t, y, x) \geq 1 + r.$$

The interest rate is only credible if $\rho_j(\mu_t, y, x)$ is such that $R_j(\mu_t, y, x) \geq 1 + r$. If $R_j(\mu_t, y, x) < 1 + r$, entrepreneurs that receive a signal x know that lender j will renege on the interest rate $\rho_j(\mu_t, y, x)$ once he observes a signal x.

A.4.1. Equilibrium

The opportunity cost of a starting a new venture in the model without capital is w, the cost of labor. With the introduction of capital, the opportunity cost of a venture is now $1 + r + \tilde{w}$, the cost of labor plus capital. If $w = 1 + r + \tilde{w}$, there is an equilibrium in the model with capital that features the same investment strategies for the entrepreneurs as in the the baseline model from Section 1.3.

The expected surplus from a venture is given by

$$S(\mu_t, y, x, \eta) = v\int_{x-\varepsilon}^{x+\varepsilon}(1-\theta)\phi(\theta|x)d\theta + \bar{\delta}(\mu_t, y)\int_{[x-\varepsilon, x+\varepsilon]\cap A(\mu_t, y, \eta)}(1-\theta)\phi(\theta|x)d\theta - (1+r+\tilde{w}), \tag{A.20}$$

which is the venture's expected payoff given (μ_t, y, x, η), minus the opportunity cost of capital and labor. Consider the following strategy for lenders: if $S(\mu_t, y, x, \eta) \geq 0$, lender j sets $\rho_j(\mu_t, y, x)$ such that $R_j(\mu_t, y, x) = 1 + r$; otherwise set $\rho_j(\mu_t, y, x)$ so high that no entrepreneur would borrow from j.[12] Consider the following rule for entrepreneurs to choose a lender: if entrepreneur i decides to borrow, only choose lender j if $\rho_j(\mu_t, y, x_i)$ such that $R_j(\mu_t, y, x_i) \leq 1 + r$. The pricing strategy for lenders and the rule for borrowers are part of an equilibrium. No lender has an incentive to deviate: if j sets $\rho_j(\mu_t, y, x')$ such that $R_j(\mu_t, y, x') > 1 + r$, no entrepreneur who observes x' borrows from j; if j sets $\rho_j(\mu_t, y, x')$ such that $R_j(\mu_t, y, x') < 1 + r$, the interest rate is not credible and no entrepreneur who observes x' borrows from j. No borrower has an incentive to deviate: entrepreneur i is better off by rejecting any lender j who sets $R_j(\mu_t, y, x_i) > 1+r$, given that there are $J > 1$ lenders who are charging lower interest rates.

In such an equilibrium, after observing x_i, entrepreneur i's expected payoff from borrowing to invest is

$$\tilde{u}(\mu_t, y, x_i, \eta) = v\int_{x_i-\varepsilon}^{x_i+\varepsilon}(1-\theta)\phi(\theta|x_i)d\theta + \bar{\delta}(\mu_t, y)\int_{[x_i-\varepsilon, x_i+\varepsilon]\cap A(\bar{\delta}, \eta)}(1-\theta)\phi(\theta|x_i)d\theta - (1+r). \tag{A.21}$$

Compared to the payoff in the model without capital, given by u in equation (1.12), we have

$$\tilde{u}(\mu_t, y, x_i, \eta) = u(\bar{\delta}(\mu_t, y), x_i, \eta) - (1 + r), \qquad \text{for all } \mu_t, y, x_i, \eta.$$

Entrepreneur i invests in equilibrium if

$$\tilde{u}(\mu_t, y, x_i, \eta) \geq \tilde{w} \Leftrightarrow u(\bar{\delta}(\mu_t, y), x_i, \eta) \geq 1 + r + \tilde{w}. \tag{A.22}$$

[12] For example, $\rho_j(\mu_t, y, x) = v + 2\delta_{\max}$.

Condition (A.22) is the same as condition (2.19) when $w = 1 + r + \tilde{w}$. In this case, the entrepreneurs' equilibrium investment strategies are the same as in the model with no capital, and Proposition 4 applies, with $\bar{\delta}(\mu_t, y) = \bar{\delta}$.

The agents' expected welfare in state s is thus given by

$$\begin{aligned}\tilde{W}_s(\bar{\delta}) =& (v + \delta_s) \int_{\theta_{\min}}^{\theta^*(\bar{\delta})} (1-\theta) G(x^*(\bar{\delta}) - \theta) d\theta + v \int_{\theta^*(\bar{\delta})}^{x^*(\bar{\delta})+\varepsilon} (1-\theta) G(x^*(\bar{\delta}) - \theta) d\theta \\ &+ (1 + r + \tilde{w}) \left[\int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1 - G(x^*(\bar{\delta}) - \theta)) d\theta + \int_{x^*(\bar{\delta})+\varepsilon}^{\theta_{\max}} d\theta \right] + (J-1)(1+r).\end{aligned} \tag{A.23}$$

The welfare in the model without capital, W_s, is described in (2.22). If $w = 1 + r + \tilde{w}$, we have

$$\tilde{W}_s(\bar{\delta}) = W_s(\bar{\delta}) + (J-1)(1+r).$$

Thus, the welfare results in Section 2.4 still hold. Lemmas 12, 13, and 14 also hold for the welfare function $\tilde{W}_s$, and so does Proposition 5.

In the model with credit, there are two types of default: default is *total* if the venture fails; and default is *partial* if the payoff from a successful venture is less than $1 + \rho$. In the equilibrium above, given their beliefs, lenders are indifferent between lending of buying risk-free bonds. In the low productivity state L, when the inefficient government makes a false report $y = h$, the agents' beliefs are biased towards the high productivity state H. Lenders thus underestimate the probability of partial default, and charges interest rates that are too low. The more agents' trust the false report h, the higher is the probability of partial default in state L, and the lower is the lenders' payoff.

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